INTEGRAL INVESTMENT MEASURES, ROBUST RETIREMENT PLANNING, LARGE-SCALE MARKET INVESTMENTS, PHASE NAVIGATION FORECASTING, AND NEW CLASSES OF FINANCIAL INSTRUMENTS:

A TECHNICAL PAPER AND HISTORICAL STUDY

by

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OVERVIEW

I propose to present a type of advanced mathematical sonar to use in investment economics. The methods presented will be used to build a new class of synthetic investments, called ***robust-return temporal integral measures***, and will particularly apply to large ***major markets.*** These techniques will ***integrate*** or straddle ***over time*** in very special ways chosen by the mathematical sonar to "swim" and "resonate" through very irregular, but still cyclical and trended, major markets. The methods will be largely ***mechanical*** in nature, in that robust long-term returns can be generated by following the prescribed plan of buying and selling over decades without attention to emotions or market fluctuations. This paper will also include methods of measuring and verifying robust returns, ways of improving long-term investing forecasting and strategies, and the design of new synthetic securities of different types.

Unlike "derivatives," which are notorious for their instability, it can be proved that the investing programs and associated synthetic financial instruments to be presented are ***more, not less, stable*** than the underlying major market indices. The ***integral*** measures to be presented will provide a robust means of systematically realizing the high long-term annualized gains of major markets such as the New York stock market – while sailing through the vagaries of the often violent, extended, and irregular bull and bear market phases. This writer does not advocate the abolition of the use of "derivatives." Rather, the introduction of integral measures will serve as a useful, stabilizing, and welcome ***complement*** to existing financial instruments and techniques, and thus will increase the breadth of the total ***suite*** of plans and instruments that can be offered to investors.

THE NATURE OF MAJOR MARKETS

Major markets such as the stock market offer very good long-term returns, but reliably capturing these returns can be difficult. Price levels in these markets follow what I will call ***homeostatic trended*** patterns. The term ***homeostatis*** is borrowed and modified from biology and refers to a tendency to revert to a long-term sustainable state, but with the possibility of irregular variation around that state. People and animals do not have to eat or sleep at the same time each day or for the same length of time each day. They can postpone either of these activities for a while, but sooner or later they must eat and sleep after all; and in the long run the typical amounts of eating and sleeping will express a long-term pattern level. Applying and modifying the term from biology to use it to describe the irregular but inevitable rhythms of economics, I speak of homeostatic patterns or ***homeostatic cycles.*** A person or firm can postpone or intensify purchases or sales, production or consumption, one can pay high prices or wait for lower ones; but in the long term these behaviors, often irregular in size, length and shape (their graph or actual record), will vary around a long-term sustainable level or state, and tend to revert to that condition, with the possibility of overshooting this middle and moving to the other direction of variation.

Market price patterns are called ***trended*** if in the long run they follow an upward trend, which serves as the sustainable "base" when subtracted away, leaving a ***homeostatic cycle.*** It is this characteristic that makes these investments so attractive. However, even broad market indices also move in large waves or ***cycles*** up and down – that often require long periods of time for a complete cycle. Market prices can move far above or below their long-term trends, and continue out of balance for years at a time.

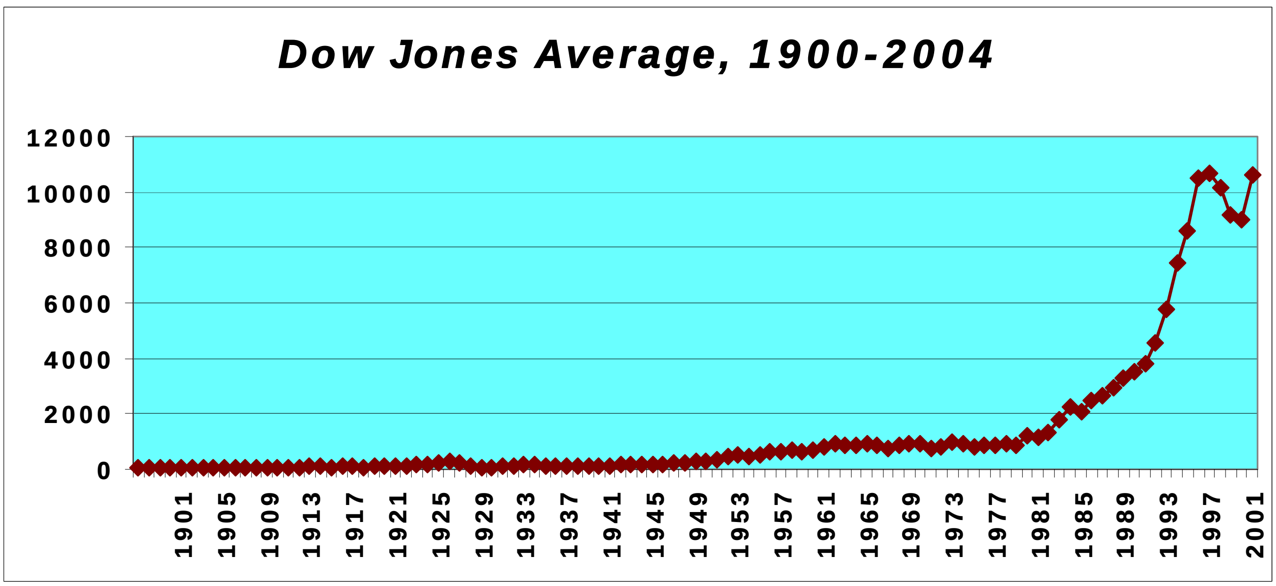
Furthermore, these market cycles do not move with the ***harmonic*** regularity of a clock! Instead, they are ***irregular*** in length, in magnitude (how far they go above or below the long-term trend), and in shape. Market prices may return to their long-term trend levels after a short time, or it may require more than a decade for normality to be restored. During bull markets, prices, even in large major markets, may reach to double or triple their long-term trend levels, causing large gains for those who sell at peak prices and severe losses for those who buy at those prices – sometimes requiring decades to overcome the damage. During recessions, market prices may fall to a fraction of their trend levels, thus presenting bargains – but many buyers are themselves depressed during such periods and are reluctant to invest.

Moreover, the market motions are irregular in ***shape.*** Prices never follow an ideal sinusoidal curve, but instead surge upwards, flatten out, and fall downwards in response to internal and external factors – often in ways that are difficult if not impossible to explain. Markets can temporarily reverse themselves, and in general be stubborn in different ways that render many investors frustrated and others insolvent.

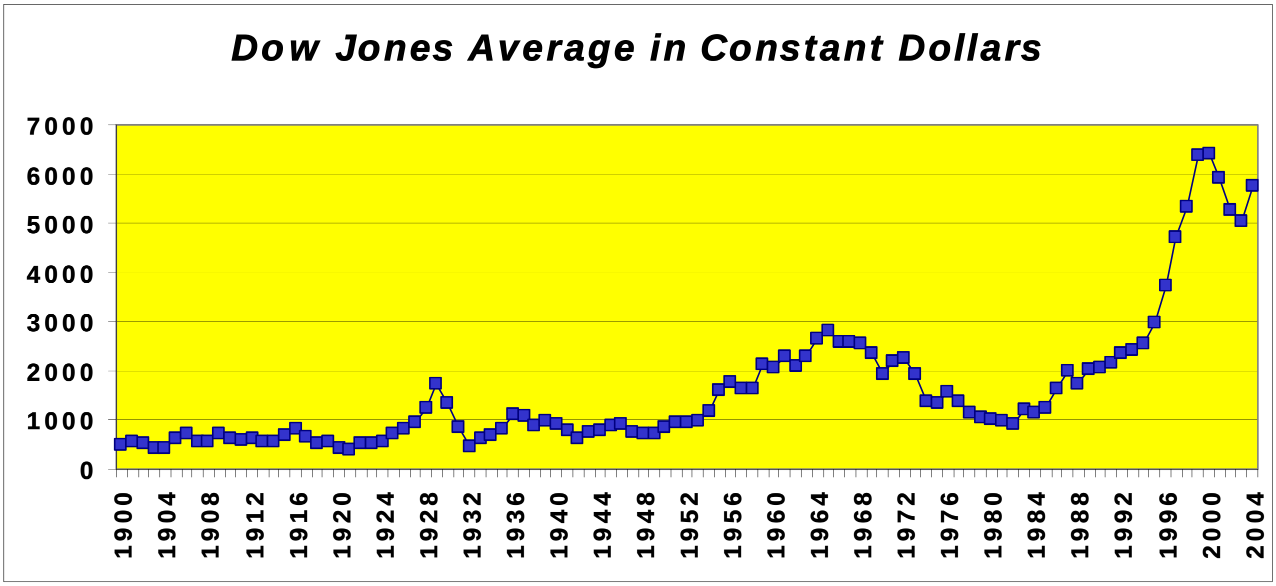
These irregular economic cycles may be called ***homeostatic,*** just as biological cycles are ***not harmonic, but homeostatic.*** Life activities such as breathing, eating and sleeping are defined in terms of typical time spans such as a minute, an hour or a day. However, a person or an animal will not take every breath exactly as the one before it. Its depth and its time span of inhalation and exhalation may be longer or shorter than normal. A person or animal may go for a longer than usual time with less than normal food or sleep – or even with no food or sleep at all. However, living organisms must go on with their rhythms of life. Sooner or later everyone must breathe again, must eat again, and must go to sleep, to wake yet again. Life goes on, around its background levels, but in a homeostatic-cyclic rather than a purely mechanical way.

The irregularity of ***homeostatic cycles,*** found even in the largest major markets, can render a cautious ***buy-and-hold*** strategy a risky one. At the very same time, homeostatic cycles give rise to ***temptations to market timing*** which promises very high returns to the wise or fortunate winners but also inflicts severe pain upon the more numerous losers – and market timing cannot be strictly reduced to an exact science.

The graph below illustrates the vagaries of even the largest markets. This study stopped at 2004 because of personal time and other constraints which precluded redoing the work through 2007, but the overall insights and conclusions are the same.



The next graph is the Dow index adjusted to ***remove the effects of inflation***, which itself can constitute a threat to investors. The index is displayed in constant dollars with 1982-1984 as the base. Here we may consider "constant dollars" as an ***alternative currency*** which serves as a truer measure of value than nominal dollar amounts.



The market has its irregularities and surprises even ***after*** inflation is removed.

Strategically, in the New York stock market, ***buy-and-hold*** isn't necessarily a very good strategy! The long-term rate of return after inflation was approximately 1.5 to 2 percent per year, not allowing for the reinvestment of dividends. However, the impact of timing is very strong. For an investment of $1,000, the best thirty-year buy-and-hold performance came to an investor who bought the Dow in 1932 and sold it in 1962, receiving $4,695 after inflation, or about 5 percent per year after inflation – but it would have taken great boldness to start such a strategy in the depression year of 1932! The worst thirty-year performance came not to an investor who bought in 1929, but to one who bought in 1955, to receive only $790 after inflation in 1985 – thus losing money if dividends were not considered. It is interesting that an investor who put $1,000 into the Dow in 1966 would receive only $1,400 in constant dollars in 1996 – not a good thirty-year return!

The variations of the market are a challenge to human fortitude. To buy the Dow in 1932 would have constituted a contrarian act of courage. As another example, investors who bought during the rising market of the 1960's and then faced heavy losses in the 1970's were unlikely to be consoled by dreams of a future bull market in the 1990's. Many of these unhappy investors simply gave up and sold out.

The long-term return is appealing, but the vagaries of market motion are distressing. The present study aims at finding a way to ***untangle the homeostatic cycles,*** to ***invert*** the vagaries of time, and recover a reliable and robust return that stands up through booms and depressions alike. That is, I propose to present a remedy for the instabilities of major markets.

The ultimate approach of ***integral investment measures*** to be developed in this study is a form of ***mechanical investing*** in that it recommends buying and selling the market in a mechanical and permanently specified way (although non-mechanical or "informed" variations on this theme will also be presented). The method by which that mechanical plan is developed is anything but arbitrary.

The integral measures presented will be ***derived from the stock (equities) market,*** but in their mathematical aspects resemble ***bonds, insurance plans, and annuities:*** in that the ***pattern of investments and the pattern of distributions is extended over time in a known and predictable manner.***

These measures emphatically do ***not*** constitute a "miracle trading" system that will systematically and substantially outpace the long-term rate of return granted by the stock market, and do it over short periods of time. In fact, these measures do not function at all over short periods of time, or even time periods of medium length! Rather, they work only as long-term projects over twenty-five, thirty, or more years. As such, they work well for retirement investment planning and as applications for large and long-lasting organizations such as pension funds, retirement funds, large investment firms, governmental agencies, and systematic add-ons to supplement existing plans such as Social Security. They also may appeal to ***sovereign wealth funds*** looking for robust returns on very large investment amounts over long periods of time without being accused of market invasion based on political or strategic grounds.

The particular advance of ***integral investment measures*** is their ***robustness.*** By ***robustness*** I mean that they reliably extract the long-term appreciation delivered by the stock market, and can be shown to do so regardless of market timing – whether the plan is begun during a boom time or a recession. The return is more robust, more reliable, than the return granted by a conventional buy-and-hold strategy. Furthermore, these plans extract a viable long-term rate of appreciation ***after inflation,*** an advantage for individuals, firms or organizations interested in retirement planning or other long-term projects.

The advance presented here is in ***robustness and dependability*** – in extracting the merit and gain that is clearly "there" in the market, and doing so ***more reliably than existing methodologies***. This gain in robustness is substantial. This paper presents integral investment plans that have ***a historically verifiable minimum (thus, robust) and usefully positive (advantageous) long-term rate of return.*** For many individuals, firms and organizations, the method of this study ***removes a great deal of uncertainty*** from investment planning. There will always be a place for short-term investment, high-risk/high-return decisions, and outright speculation; but these moves carry additional risk and insecurity along with their higher possible gains. Such insecurity is generally undesirable in retirement-related and other long-term planning; for such planning the methodology of integral investment measures may be a welcome addition to the existing financial state of the art. The net gains in efficiency and uncertainty are likely to be equivalent to many billions of dollars.

For centuries people have been buying and selling synthetic financial instruments ***derived*** from other ***underlying*** assets (financial entities having more concrete existence), such as stocks, bonds, commodities, and interest rates. These investments include futures, put and call options, swaps, and a panorama of mathematical constructions which is almost infinite in its diversity and complexity. In general, ***derivative*** investments tend to be ***more volatile*** and ***less stable*** in value than the underlying assets from which they are derived. Derivative investments are often short-term in nature and are sometimes used to ***hedge*** against possible changes in their own underlying assets. The body of past and current literature on derivative investments is immense.

Derivative investments, such as options or buying on margin, often have high leverage, so that a small investment may yield very high returns or suffer large and abrupt losses; such investments often tend to be speculative in character. The instability of derivatives has made them a mixed blessing to the investing community.

In the language of calculus, an ***integral*** sums up the values of a mathematical function, and works in the opposite direction from a ***derivative.*** Investors often diversify or mathematically ***integrate*** their portfolios through the use of ***mutual funds,*** dividing their money among more than one stock or even more than one market sector. In fact, major market indices such as the Dow Jones Industrial Average or the Standard & Poor 500 index represent weighted averages or ***integrals*** of the stock prices of the leading companies in the nation. People are now able to invest in ***index funds*** which track the behavior of these major market indices. The attractiveness of these investments is that they are ***more stable*** than their individual component assets. It is also now possible to buy and sell instruments ***derived*** from integral-type market indices, such as futures based on a major stock market index.

The body of published and unpublished literature on these investments is considerable, although I believe the use of the mathematical term "integral" to understand them is my own.

The present paper makes the distinctive contribution of designing investment plans that are ***integrals over time*** as well as integrals over a set of corporate equities. The plans suggested here are not the elementary time-plans such as "buy and hold." Rather, they are built from a special set of weights or

measures over time, which are themselves constructed from a particular mathematical "radar" or "sonar" that extracts the salient homeostatic-cyclic features of the market – and inverts these features for long-term robustness and stability. The word "measures" was inspired by the "measure theory" and "Lesbegue measure" presented in graduate level mathematics as a deeper generalization and extension of calculus.

We can understand these products better by understanding their ***dimensional*** characteristics. What is meant here by "dimensionality"? In physics, a meter is a unit of length while a second is a unit of time. Velocity (speed) is described in terms of distance covered over time – in units such as miles per hour or meters per second – literally, distance divided by time. Reasoning in the reverse direction, velocity added up (integrated!) over time results in distance covered. As another example, energy is measured in units such as ergs or joules. The discharge of energy over time is called "power" and is dimensionally described in terms of energy divided by time. Reasoning in the reverse direction, power stored or accumulated (integrated!) over time results in a build-up of energy.

Economics has its own terms of dimensionality. ***Stock*** variables, such as the amount of money in an account or the price of an asset such as an equity share or a mortgage valued on the secondary market, are distinguished from ***flow*** variables, such as the rate at which money is earned or paid out per year, per day, or per hour from that account, stock, property or other asset. Of course, the relationship and association between the flow generated by an asset such as a bond or a mortgage and its market price is expressed in terms of a ***rate of interest*** (as well as other aspects such as the expected probability of default, which leads to an appropriate premium or discount to express this risk).

The value of an equity share, a bond or another asset is measured in dollars as a "stock" variable. Its dividend yield over time, or other system of payouts (as distinguished from appreciation) is a "flow" variable, measured in dollars over time. As mentioned, the relationship between flow and price is mediated by the term "interest rate" which can associate a flow of payout with a corresponding price, and vice versa.

A ***derivative*** financial instrument can have its own distinct dimensionality. It may be expressed or ***derived*** in terms of the rate of change of an underlying asset over a period of time (analogous to velocity) or in terms of a difference or other relationship between one price and another, or one interest rate and another, expressed in complicated and subtle ways. It may be defined in terms of options to buy or sell, again expressed in mathematically complicated and subtle ways. The variety and dimensionality of the universe of derivatives is immense.

The dimensionality of an integral plan is properly measured in terms of price multiplied by time, price both accumulated and dispersed over time, in the same way that units of velocity multiplied by time yields units of distance and units of power multiplied by time yields units of energy. As such, integral plans literally represent a new dimension in finance, especially when understood in their non-elementary designs. The actual distributions of buying and selling over time used by an integral investment plan are called ***integral investment measures.*** These measures belong to this new dimension/dimensionality in economic units.

There is clearly a virtually infinite diversity of possible integral buying/selling distributions and measures that can be used in buying and selling – whether ***mechanical*** in definition or ***informed*** by market conditions, and whether the distribution of buying matches the distribution of selling or not.

It will be demonstrated that a special and particular subset of integral investment measures, derived by the non-elementary, novel, non-obvious and distinctive methods presented here, historically yield a high long-term rate of return, and do so in a robust way that removes a great deal of uncertainty from long-term and planning.

THE DISTINCTIVE CONTRIBUTION OF THIS STUDY

The new contribution of any paper or study is that part of its invention or findings which is "novel and non-obvious," meaning that it goes beyond and is different from what appears in the existing body of known or "prior art," and is not obvious to one acquainted with that existing body of information. This paper makes several distinctive and new contributions which go beyond the current body of knowledge, whether contributed by others or by this writer himself.

The present paper systematically and mathematically addresses the issue of advanced ***measures*** of investment, ***distributed and integrated over time.*** Yes, there do exist simplistic plans consisting of general advice such as "buy and hold for a long time," "put in a certain amount every month and leave it there," "average your dollar costs," "buy the index," or "diversify your portfolio and rebalance it from time to time." This paper presents systematic and thorough mathematical constructions of ***integral measures over time*** built according to advanced mathematical techniques, and having certain desirable properties such as ***robust return*** (a historically verifiable minimum and usefully positive return).

The ***integral measures*** approach devised by this author has the particular advantage of having novel and non-obvious ***emergent properties.*** In science, emergent properties or characteristics are phenomena which arise in composite and complex situations, which are not obvious or immediately deducible from the rules which govern their elementary components; although of course they are not in direct contradiction to them. A large body of literature and research has been built on the subject of "complex systems" and "chaos," showing a lush variety of phenomena and properties that come as a surprise to any reader. These emergent properties are not directly apparent from the consideration of the elementary equations governing the fundamental particles which make up the system.

To give other examples, one cannot imagine normal human beings deducing the design, construction and capabilities of a modern computer processor directly and immediately from the laws of particle physics or the basic properties of the chemical elements. These capabilities, both physical and non-physical (in the sense of being able to execute software commands and thus emulate the thinking process, but many times faster and far more accurately than a human could do), are not ***inconsistent*** with the laws of physics or with elementary chemistry – but they are not, at least for human beings, immediate and obvious deductions from those fundamental laws and properties. For this reason, it was and is entirely appropriate to consider the design and construction of a computer chip as a novel and non-obvious invention.

One important ***emergent property*** of the integral investment measures described by the present paper is that of ***robust return*** – that is, ***a minimum and positive return*** over time, which can be and has been verified across many years of data, including both boom times and severe depressions. In other words, these measures can be said to reliably capture the excellent long-term returns in major markets. Such a property has tended to elude prior research. It is certainly not an obvious consequence of existing methods of investment or existing "derivative" instruments.

Another important ***emergent property*** is that the integral investment measures constructed by this work can be shown to be ***more, not less, stable*** than their underlying market index or asset value. In view of the known instability of ***derivatives,*** such stability would be welcomed by many investors and planners, especially those interested in retirement investment or otherwise taking the long view. This emergent property is not an obvious consequence of any prior definition or construction, nor is it an immediately perceived consequence of the elementary rules and practices of investment.

The published work of this author ***has*** developed some simple forms of integral investments. However, it must be said that the proposals and methods described in this paper include material which is novel and non-obvious ***even relative to my own previous work.*** This paper constructs integral measures which are much more sophisticated than in my previous work and have new aspects of development, enabling a wide spread of options in constructing such measures. The present paper also includes a ***mathematical definition of robust return,*** and a corresponding mathematical test to ***verify*** if a robust return can be said to exist for a user-suggested strategy of measured investment.

The present paper also includes a form of ***mathematical sonar*** used to ***build satisfactory integral investment measures*** from component mathematical functions and operators. My own previous work included a limited form of this sonar, and constructed only one type of integral investment measure. The present study expands my previous work into new dimensions and provides a deeper and more thorough mathematical sophistication. It makes possible the construction of many new forms of integral investment measures tailored to the needs and expectations of investors.

My previous work included the construction of simple ***phase diagrams*** and developed a ***Phase Method*** of projecting the long-term price behavior of major markets. My published papers projected the bull market of the 1990's years in advance and also warned late in the 1990's that the then current market behavior was unsustainable. The number of analysts and market researchers who correctly and publicly called ***both*** the bull and bear phases is quite small – probably less than ten and perhaps less than five. Please consider the following published statements:

> "A Dow of 4000 to 6000 (before allowing for inflation) would not be impossible by 1994 to 1996." ("The Phase Method of Analyzing Economic Cycles" by Christopher Cagan, published in ***Cycles*** magazine, July/August 1991, research done and paper submitted to the journal in 1990).
>
> "What about all the profits being made in the current bull market? In most cases, they are simply being made on paper – or in a file on a computer disk…The only way to take advantage of these profits is to take money out of the market and [invest it] elsewhere…Otherwise, many could experience a rise from rags to riches on paper only, not receiving their hoped-for profits when they sell." ("The Phase Method: 1998" by Christopher Cagan, published May 1998 in ***Technical Analysis of Stocks & Commodities*** magazine.)

I do not know of any other writer who built such economic phase diagrams. The present paper builds upon my prior work, using the Phase Method to modify the integral investment measures constructed by the mathematical "sonar," in order to improve net returns.

Another aspect of the present paper is the construction of many varieties of ***synthetic instruments or securities*** inspired by the ***integral measures*** approach. These financial instruments can be constructed for almost any major market, not merely the New York stock market. They can follow the long time scale appropriate to integral investment measures, or they may not do so. They can take on a wide variety of mathematical forms and may themselves be traded. In general they are quite stable. In turn, they can generate and define many instruments and securities which are in turn ***derived*** from them, including futures, options, puts, calls, swaps, and many other types of constructions. These "new derivatives" would be less stable than synthetic integral measures but more stable than existing "derivatives."

It should also be remembered that the integral measure investment functions described in this paper do not ***need*** to be packaged or defined as separate securities of their own – although they ***can*** be so defined, as the preceding paragraphs have stated. They can also be defined as investment ***plans*** of buying and selling over time. When these measure functions are ***not*** packaged as separate securities they avoid any problem of "tapping out" that might arise when the measure function (or a hypothetical synthetic security) pulls ahead in value or falls behind in value relative to its basic index such as the Dow. On the other hand, some may ***prefer*** to define synthetic instruments anyway – or even ***define*** new securities built around the ***difference*** between an integral measure-based security and its underlying Dow (or other) index.

In summary, the present paper includes several novel and non-obvious features.

A FIRST LOOK AT THE APPLICATIONS

Before going into a survey of the mathematics involved in integral investment measures, it is appropriate to mention some possible applications.

First, the integral measures constructed in this paper, with their robust returns, are very useful and attractive for investors interested in the long term. These methods are not intended for day-trading or short term speculation. These investment measures do not double in a single day or even a single year – but neither does their value collapse! On the long term, their performance is excellent, and they reliably capture the attractive long-term annualized return associated with the stock market, straddling the inevitable bull and bear cycles. The approach of these integral measures can also be applied to national income, commodity prices, and almost any major market which exhibits homeostatic trended behavior. They may be applied to the basic price data of these markets, or to modified forms of these data series – perhaps adjusted for inflation or for seasonal variation – or in some other way.

Particular applications of these investment measures include ***retirement fund investment*** and even ***add-ons to Social Security.*** Many Wall Street firms would be interested to know that there is a safe way of investing for retirement. Many political leaders would also be interested in the findings of this present study. Perhaps also, ***sovereign wealth funds*** might be interested in a diversified way of making large and long-term investments to get a robust return, without creating upheavals in the markets in which they invest, and investing in a broad and passive way so as to avoid accusations of political and strategic interference.

Second, the work described in this paper provides reasonable long-term ***navigation and forecasting*** for major markets. The ***Phase Method*** described here does not forecast short-term market swings, but it works reasonably well in evaluating long-term business cycles and suggesting how these cycles may develop from the current market position.

Third, the work described in this paper describes many ***new synthetic financial instruments*** which themselves may be bought, sold, traded, and hedged; and yet other instruments may be "derived" from them, in total constituting a very large set of applications. These instruments may be defined in normal units of time or with respect to an abstract "phase time" – which has special and desirable properties. The investing community is always interested in new financial structures and instruments, not only to advertise and trade, but because these new instruments may be suited to the needs and concerns of investors and firms.

Fourth, these integral investment measures are attractive because they are ***more, not less, stable*** than their underlying indices such as the Dow Jones average or HPI indices of real estate prices. In this feature they avoid the notorious ***instability*** of existing derivatives which has brought sadness to many – and threatens to bring losses to others in the extremely large derivative market.

In particular, with the financial community smarting under various failures, scandals and accusations, it seems reasonable to me that some would be interested in studying and marketing financial instruments possessing stability rather than volatility.

Fifth, the stability and long-term advantages of integral measure instruments may be used to do good for the people of our country! Many are worried about their retirement and would welcome the arrival of stable investment plans.

Sixth, the methods and approaches of this work reflect the virtues of solidity and integrity that have characterized the better features of American business for centuries. The method reflects the philosophy of steady investment for the long run, and avoids the instability of speculation and of existing "derivatives." In fact, it suggests investment approaches that are more, not less, stable than the underlying major market index from which they are built. To publicize the stable investments proposed here would make a contribution to our society in contrast to tearing something from it.

INTEGRAL INVESTMENT MEASURES AND ROBUST RETURN: INTRODUCTORY MATHEMATICAL CONSIDERATIONS

This part of the paper describes some of the key mathematical definitions and methods of the study. Please excuse the technical language and try to read around it if you prefer to do so.

An ***integral investment measure*** is a pattern of purchases of a major market index over time, defined by a function f(t) where t represents time and f represents a mathematical function. Just as an integral in calculus represents the area under a curve, so the total quantity of "units of the index" purchased is given by the integral $\int f(t)dt$. If the purchasing is done at discrete intervals, the integral may be replaced by a summation $\Sigma f(t)$.

To study the distribution itself, f(t) may be multiplied by an appropriate number so as to make the total area under the curve equal to 1, just as in a probability distribution in statistics. In this case, f(t) represents the ***proportion*** of the total investment, measured in terms of money or in terms of assets purchased, that is actually invested at time t. In the preferred form of this understanding we will usually assume that the total area under the curve f(t) is equal to 1.

This money invested is used to purchase an index (or its components such as stocks) whose price at time t is given by the function m(t). This price series consists not of a theoretical function but of ***empirical data*** – of actual price levels such as the HPI real estate price index for a metropolitan area, or the Dow Jones Industrial Average, or any other major market data series that is homeostatic and trended. This data series may represent literal prices, or it may represent prices corrected for inflation (which is the preferred version), or it may represent prices modified in some other way.

It is possible to combine both continuous purchases and discrete "lumped" purchases into one single formula using an advanced form of mathematics called Lesbegue measure – but that will not be developed in this present paper. Instead we will develop the mathematics for continuous and discrete functions separately.

In actual practice, no investment program continues forever. It has a beginning and an end. If the time when the investments begin – the first time "t" for which f(t) is greater than zero (or "takes off from zero") is called t_0, and the investment program is carried out over an elapsed time "p", then the investment program ceases at a time t_1 defined by $t_1 = t_0 + p$.

The total investment is $I = \int f(t)m(t)\,dt$.

If investments are made in lump sums, we have $I = \sum f(t)m(t)$.

Now, if investments are to benefit the investor, they must be sold and money extracted. Thus, we must postulate a distribution of ***sales*** as well as a distribution of ***purchases.***

The preferred form of the measures developed in this study uses what I will call a ***push-pull*** arrangement, defined as one as where ***the distribution of sales is exactly the same as the distribution of purchases, but shifted forward by a time interval called "s."*** Mathematically, the sales function is f(t-s). The sales begin at t_0+s and end at $t_0 + p + s = t_1 + s$. [More advanced forms may have two different distributions; examples will be given later; but the push-pull plan will constitute the preferred form of this development.]

The entire plan is defined by three numbers and one mathematical function:

The starting time t_0.
The total purchasing (and selling) elapsed time p.
The shift s between the purchases and the sales.
The function f(t). This function can be of many shapes and forms; but it must be positive
if $t > t_0$ and if $t < t_1 = t_0 + p$, and zero if $t < t_0$ and if $t > t_1 = t_0 + p$;
and the total area under f(t) from t_0 to $t_0 + p$ must equal 1 in the preferred form.

Strictly speaking, we could dispense with "p" and speak only of the definition of the function f. However, the development of the work will proceed more easily and more understandably if we also include "p." It will also make it easier and more natural, in that we can use standard functions "f" with standard formulas and parameters, modified with various values of p. For instance, we could speak of a flat buying plan (f is a constant) spread over a time span p, or a function f that follows a standard mathematical pattern (triangle, sine curve, polygon, polynomial, etc.) spread over a long or short time p.

The plan is rigidly defined in advance without reference to news events or the market prices m(t). As such, it may be called a ***mechanical*** plan. Once set in motion it requires no further decision on behalf of the investor. This is the opposite of a "market timing" plan, but is well suited to long-term retirement investing plans, especially those set up for large numbers of people of differing ages. In fact, this plan may actually function better for large collective groups of different ages (or companies or governmental programs that work with or on behalf of such groups) than for a single individual.

In simple language, what is happening here? Purchases are made in units of the major market index, according to the function f(t), beginning at the starting time t_0 and ending at time t_1. Then, shifted over by a fixed time "s" later, sales are made according to the same function, beginning at $t_0 + s$ and ending at $t_1 + s$. If s is small the two distributions, of purchases and sales, will overlap. If s is larger than p, the purchases will conclude before the sales begin.

Another way to say this is that each separate purchase of the index is held by the investor and sold according to plan a fixed time "s" later, regardless of whether this sale incurs a profit or a loss!

Depending on the value of f(t), a purchase made at time t [whether this is done continuously or at discrete intervals in "lumps"] may constitute a larger or smaller number of units of the underlying index. In such a case f(t) is measured in terms of the units purchased, not the money expended to purchase them. The expense will be f(t)m(t) where m is the market price of the index; this will be measured in money.

When this same slice or pulse is mechanically sold after an elapsed time "s", at time t + s, the sale in units will be the same f(t) and the money returned to the investor will be f(t)m(t+s). The net profit or loss will be

$$f(t)m(t+s) - f(t)m(t) \quad = \quad f(t)*[m(t+s) - m(t)].$$

There will be a profit on this slice if m(t+s) > m(t), that is, if the market has gone up between time t and time t+s.

There will be a loss on this slice if m(t+s) < m(t); that is, if the market has gone down between time t and time t+s.

The percentage of the profit or loss depends on the ***amount of rise or fall*** of the market index, and the actual monetary gain or loss also depends on how big an investment was made – how ***many of the units*** of the index – the investor bought and later sold.

Inevitably, some slices or pulses will make a profit when they are sold, and some will result in a loss. If the market index is ***trended upwards,*** there will be a bias in favor of profit, especially as the holding period "s" increases. But how may an overall net profit be ***reliably*** – in other words, ***robustly*** – secured?

The key task – and key insight – of this study is to develop appropriate choices of the function f so as to reliably secure a ***robust return.*** That is, the total overall return should be positive (and as large as possible), ***regardless of what time t_0 is chosen to begin the plan, and regardless of the choice of the holding time s!*** If this were possible, then an investor could begun such a robust plan at any time, whether the market was currently in boom or depression, and could make the holding time large or small, and still reasonably expect a positive return. The key point is to ***find*** such a function that straddles the irregular up and down cycles, and captures the long-term positive trend.

What is the total return from ***all*** the slices of buying and selling?

As an integral, that is $\int f(t)*[m(t+s) - m(t)]\, dt = \int f(t)*m(t+s)\, dt - \int f(t)*m(t)\, dt$.

As a sum, that is Σ f(t)*[m(t+s) – m(t)] = Σ f(t)*m(t+s) – Σ f(t)*m(t) .

The integral or sum in the two cases runs from t_0 to $t_0 + p$.

By a simple mathematical substitution, this may be expressed as

$$\int f(t-s)*m(t)\,dt \;-\; \int f(t)*m(t)\,dt \quad \text{or}$$

$$\Sigma\, f(t-s)*m(t) \;-\; \Sigma\, f(t)*m(t)\ .$$

Here time in the ***first*** integral or sum runs from $t_0 + s$ to $t_0 + p + s$, and time in the ***second*** integral or sum runs from t_0 to $t_0 + p$.

In multiplicative terms, the total ratio of return to investment is

$$\frac{\int f(t-s)*m(t)\,dt}{\int f(t)*m(t)\,dt} \quad \text{or} \quad \frac{\Sigma\, f(t-s)*m(t)}{\Sigma\, f(t)*m(t)}$$

where time in the numerator runs from $t_0 + s$ to $t_0 + p + s$, and time in the denominator runs from t_0 to $t_0 + p$.

As an annualized quantity, the rate of return is

$$(1/s)*\mathrm{LN}\left\{\frac{\int f(t-s)*m(t)\,dt}{\int f(t)*m(t)\,dt}\right\} \quad \text{or} \quad (1/s)*\mathrm{LN}\left\{\frac{\Sigma\, f(t-s)*m(t)}{\Sigma\, f(t)*m(t)}\right\}\ .$$

where time in the numerator runs from $t_0 + s$ to $t_0 + p + s$, and time in the denominator runs from t_0 to $t_0 + p$, and LN represents the natural logarithm. This annualized return ideally should be positive ***regardless*** of the starting time t_0 and the holding time s.

A necessary and sufficient condition (hence an equivalent condition) for this happy situation is if

$\int$ f(t)*m(t) dt or the corresponding Σ f(t)*m(t) (time from t_0 to $t_0 + p$ in both cases)

always increases as t_0 increases – thinking of t_0 as a variable subject to change rather than a constant. In such a case the amount realized from sales will always be larger than the amount paid in purchases.

This will be defined as the ***robust return*** condition. In such a case, the numerators of the two equations above will always be greater than the denominators (regardless of the starting time t_0 or holding time s), and the overall annualized return will be positive. We have a ***robust minimum return*** if the rate of increase is always equal to or greater than a positive number which is the minimum rate of return.

In actual practice we should seek to set up the function f and its time span p (and the holding time s) so as to get a ***minimum*** annualized return as large as possible. For instance, it would be nice to get a ***minimum, hence robust,*** annualized return of 5% to 6% ***after*** inflation, equivalent to 7% to 10% annually ***including*** inflation, across 100 years of positive and negative market history and starting times. Such a return, if it was ***robust*** and held up across all phases of market history and cycles, would be attractive for retirement investment plans that spanned several decades.

Since the market data m(t) are inputted externally (***empirically, from outside***), there is no easy formula that will always define f so as to generate a good minimum robust return. Instead, f must be built ***experimentally.*** It will be ***essential*** to use market data m(t) from a long time series – certainly longer than one approximate (homeostatic, bottom-to-bottom or top-to-top) cycle, and ideally including two, three, or more market cycles. Having this data, we will experiment with various functions "f" until we find one producing a satisfactory minimum robust return, and that "makes sense" in that it fits with the overall theme of homeostatic trended cycles.

INTEGRAL INVESTMENT MEASURES AND ROBUST RETURN: AN OVERVIEW OF THE METHOLOGY

How should we ***build*** this function f that defines an integral investment measure function? That is the province of the ***mathematical sonar*** mentioned earlier in the paper.

It is well known that any function meeting certain basic conditions [generally easy to meet] may be expressed as a sum of a standard series of ***component functions.*** This series must be infinite in its size to cover all possible functions, but in many cases only the first few components of the total set need be used to provide a good approximation. It is easiest if these components are mathematically ***orthogonal*** (not collinear or correlated) but strictly speaking that is not required.

The best known arrangement in the mathematical literature is called ***Fourier analysis,*** named after the mathematician Fourier. This analysis breaks down a function as a linear combination (weighted sum) of traditional sine waves of different lengths (or frequencies). As more and more waves are allowed, the approximation becomes closer and closer until even an irregular or sharp-pointed function can be closely modeled.

The ***mathematical sonar*** of this paper will work its way through a different set of component functions, which are in turn tested against the empirical market price data m(t), until a satisfactory robust-minimum-return function is experimentally constructed and verified. A ***distinctive contribution*** is that this approach will not begin with a standard set of Fourier sine waves, but instead with a different and more varied set of standard functions, suited to homeostatic trended economic cycles. The Fourier waves can still be used, but the set of component functions is not limited to them.

This method will only be practically useful if the market prices m(t) are in fact positively trended. If there is no upward long-term trend, one should not expect a function f(t) to systematically yield a robust minimum positive return. For instance, in the case of a ***neutral*** random-walk series (as distinct from a random walk superimposed on an upward trend) there is no robust minimum return and all investing "systems" will tend to a long-term return of zero. Any gains will be made by accident and there is no guarantee that they will be repeated! If the series is ***negatively trended*** (such as a random-walk superimposed upon a house percentage, as is the situation that is found in gaming houses), the results will be even worse.

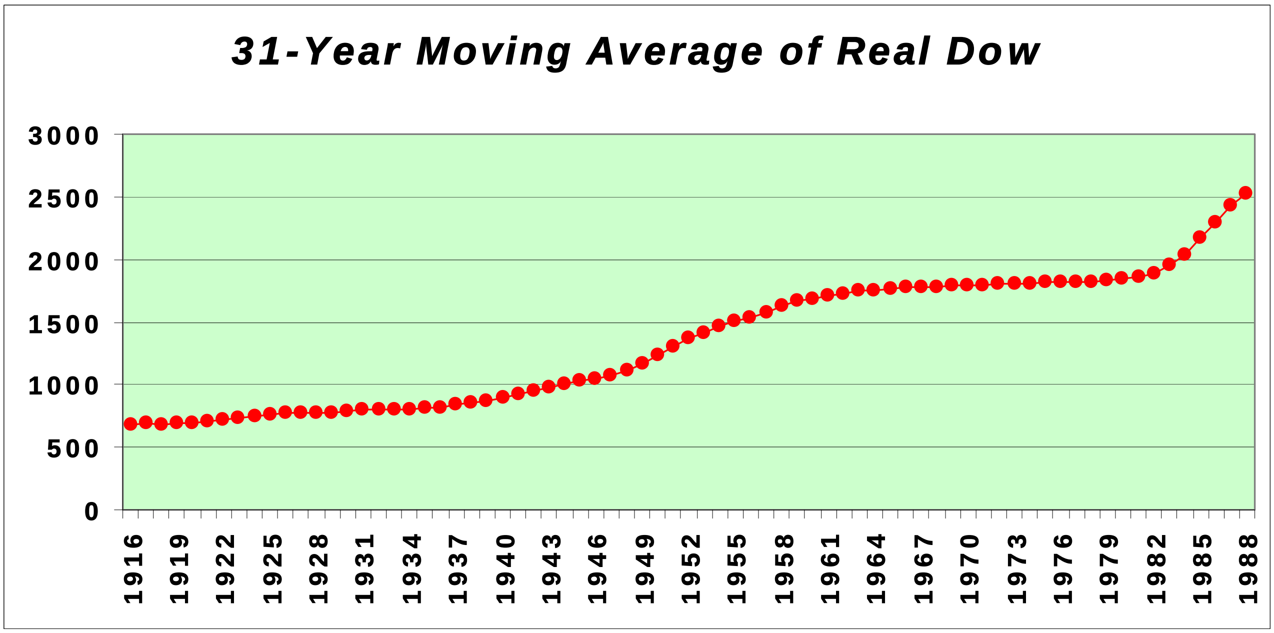
What sort of component functions may be used? The simplest purchase plan is a one-time purchase sold a fixed time "s" later. In such a case the number "p" does not arise since there is only one single purchase followed by one single sale. Such a plan will have a robust return if and only if the market m(t) always rises! If sometimes the market falls, the return will be negative.

The situation might be improved by introducing restrictions on the holding time "s". This is the mathematical definition of the classic buy-and-hold strategy. However, as we have seen, it is possible to buy and hold the Dow Jones Industrial Average for 30 years (s = 30) and still lose money after inflation, if the plan is begun at the wrong time t_0. We have seen that $t_0 = 1955$ resulted in a badly negative return after inflation. This danger can tempt an investor into market timing, which has its rewards and also its risks.

Another issue here is the conflict of psychology and economics. We have seen in hindsight that $t_0 = 1932$ would have been a good time to begin a thirty-year buy-and-hold strategy – but to invest in the market in 1932 would have required incredible contrarian courage. In general, the numbers can "play with the head" of an individual investor, or the investing community in general, which is prone to a "herd mentality," and produce either irrational exuberance or profound psychological discouragement, in each case leading to unwise investment decisions.

To get a robust return in the Dow after inflation (not counting dividends) requires a much longer time span than thirty years. For instance, purchasing the Dow between 1909 and 1912 and holding it for ***seventy years*** until 1979 to 1982 would produce an annualized return after inflation of less than ***one percent, often closer to one-half percent.*** Since 70 years far exceeds the investing lifetime of almost everyone, we conclude that buy-and-hold is a strategy that needs to improve.

A ***flat*** function f of buying and selling will do better than a single buy-and-hold transaction. This would involve buying a fixed number of "index units" (say of the Dow) every year for a fixed number of years, and then selling these purchases one at a time a predetermined number of years later [in fact it is now no longer necessary to buy the Dow stocks but simply an ***index fund***]. This is mathematically equivalent to taking a ***moving average*** of the index and seeing how that performs. However, it is very hard to get a good robust minimum return. Even a thirty-year moving average for the Dow has its slow, almost flat, spots. Although the decline is too small to see, this average actually went down from 1925 to 1926. The graph below, using a 31-year average, comes from the Excel file "FILE 1A FULL EXTENDED TRIANGLE DISTRIBUTION with graphs."



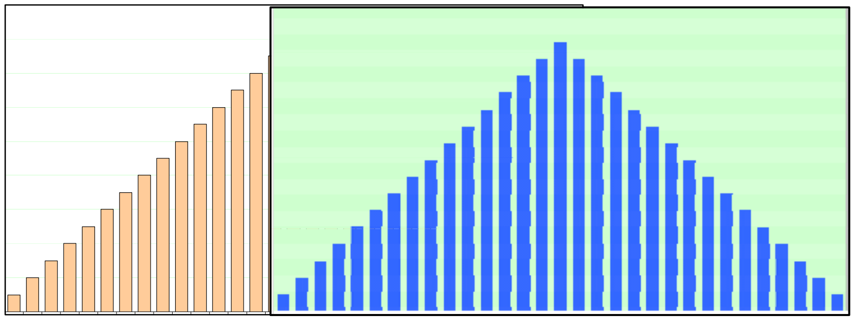
The real improvement comes in using a ***triangular, double-smoothed moving average*** function for buying and selling – the moving average ***of*** the moving average. Imagine an investor

> Buying 1/961 unit of the Dow in 1930 and selling it in 1935, and
> Buying 2/961 units of the Dow in 1931 and selling it in 1936, and
> Buying 3/961 units of the Dow in 1932 and selling it in 1937, and…
> Buying 31/961 units of the Dow in 1960 and selling it in 1965, and
> Buying 30/961 units of the Dow in 1961 and selling it in 1966, and…
> Buying 2/961 units of the Dow in 1989 and selling it in 1994, and
> Buying 1/961 unit of the Dow in 1990 and selling it in 1995. (Note 1+2+…+31+30+…+2+1 = 961.)

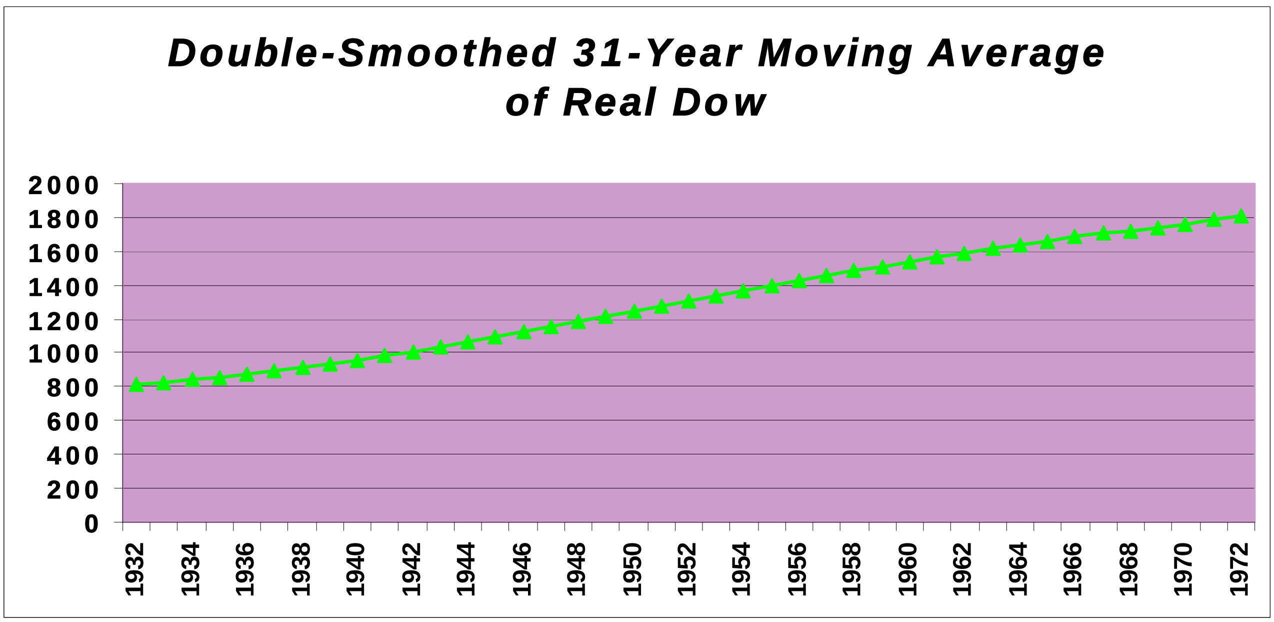
The investor does ***not*** need to think about the "constant-dollar Dow." He merely buys the appropriate number of units of the Dow ***itself*** at the ***then-current*** prices, and sells it later at the end of the investment span, at the market price at ***that*** time. But in constructing the plan, we have already taken inflation into account (all that is ***built into the background***), and the investor ***comes out with*** a good return ***after*** inflation.

Here the investment span "s" is 5 years. To change "s" to six years instead of five, replace 1935 by 1936, (etc.) leaving 1930 (etc.) unchanged. In the plan given above, the selling triangle lags the buying triangle by the investment span of five years.

The pictures below represent an informal attempt to show what is going on. The purchases, spread over 61 years, are in tan. The purchase triangle is partly obscured by the selling triangle, which is shown in blue. In the visuals below, the investment span, the time "s" between the peaks of the two triangles is about 28 years!



Those who are mathematically inclined will see that this triangular weighting pattern exactly tracks the ***double-smoothed moving average*** (the moving average ***of*** the moving average) of the Dow after inflation. That curve is graphed below. It also comes from the Excel spreadsheet "FILE 1A."



Here the index is graphed in constant 1982-1984 dollars. The graph terminated after 1973 because the horizontal axis represents the ***center*** of the triangle, and the triangle needs 30 years after the center time to be complete.

The virtue of this approach is clear: ***the graph always rises!*** This exactly corresponds to the ***robust return condition.*** Moreover, the rise in the graph is fairly stable!

Experimental research has shown that this method ***does*** give a good robust minimum return. The author's paper "Integral Investing Patterns: A Historical Study" (2000) studied the period 1900-1996 and left out the extreme bull market of the late 1990s. For s = 10 years, s = 20 years, and s = 30 years the mean and median annualized rate of return was 2.0% to 2.1% and the ***worst*** annualized rate of return was 1.5%.

This is 1.5% per year, ***after*** inflation, and ***without*** allowing for the reinvestment of dividends. The consistency of the return was impressive; the standard deviation of the annualized rate of return was typically only 0.3% to 0.4%. In summary, the return provided by this plan is ***far more stable*** than that of the original index, which itself is a weighted average of 30 major stocks.

This method captures the attractive long-term return of the stock market while removing a great deal of uncertainty. A 2% return after inflation without dividends, after adding 2% to 4% for dividends (whether reinvested or not) and including 2% to 4% for inflation, yields an annualized current-dollar return of 6% to

10% – which is an ***excellent*** long-term, ***low-risk*** rate of return! Admittedly, this is not a short-term trading system or even a middle-term plan, but does work on a long-term basis and is appropriate for a retirement plan.

This triangular arrangement also has some personal demographic advantages: the years of peak investment are typically at the middle age of working life, when savings are greatest.

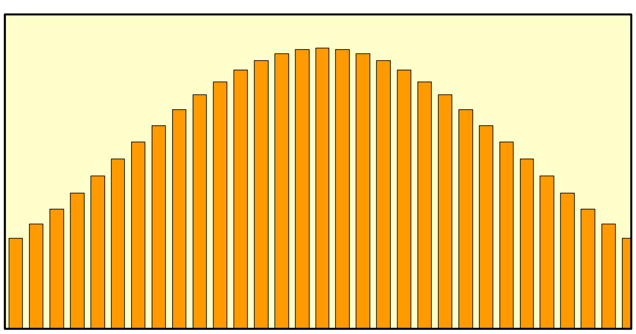
At this point it is appropriate to answer some questions. First, why talk so much about thirty years? I have experimentally found that even double-smoothed moving averages for shorter times such as ten or fifteen years do not work well for the Dow. From the graph on page 2, one sees that the homeostatic cycles of the stock market are "about" 30 to 40 years long. Consider the years of market peaks: 1929, 1966, 2000.

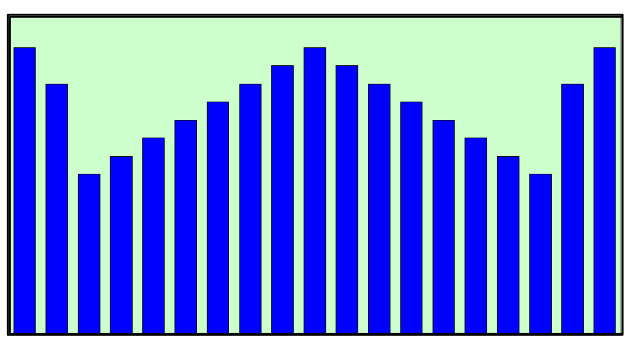
In prior work I did a Fourier analysis, including a spectral analysis and a periodogram. The results yielded a resonance at about 30 to 33 years. Exact cycle lengths are ***not*** required. This system does not require cycles to be elegant in form, equal in size or shape, or equal in length – only that very approximate homeostatic cycles exist around a long-term upward trend. As we have seen above, a double-smoothed 31-year moving average, corresponding to a triangular buying and selling distribution, works well. If a particular homeostatic cycle turns out to be 26 years long, or 40 years long, the double-smoothed 31-year moving average will still work reasonably well. Robustness is an advantage in real-world situations.

Strictly speaking, these triangles take over 60 years to begin and end. However, in practice we can trim off the tips of the triangles and consolidate that weight elsewhere, since the tips contribute only a small weight to the distribution. There will be some sacrifice of the level of minimum return, but it is possible to shorten the total time needed while keeping the minimum return above zero (no losses). Please see the supporting Excel files for further information.

In the language of this paper, a ***positive robust minimum return*** for the Dow after inflation is only possible for about p = 30 years or longer. This is why the present model provides no help for day-traders, speculators, or even middle-term investors looking to make money within a few years. It has substantial value, however, to long-term investors looking to meet their retirement needs or the needs of their children, as well as to large mutual funds, state or private pension funds, or a government-regulated add-on program for of Social Security. The "mechanical" or "blind" nature of these plans makes them especially appropriate for Social Security add-ons or pension funds because there is no room for accusations of bias or the temporary pull of expediency or temptation or political pressure. The system functions essentially by itself.

Continuing the process of mathematical sonar, there are many functions "f" that can be tried beyond simple lines and triangles. One might wish to consider quadratic, cubic, and higher polynomials. However, mathematical knowledge would suggest the use of refined functions that are themselves approximately cyclic and lend themselves naturally to "fit" homeostatic cycles, including sine and cosine functions, truncated sine and cosine functions and truncated normal distributions (the truncation is to cut the time shorter by trimming off the small tails of the distribution, at a cost in consistency and robustness). One can also experiment with "concentrated" triangles with "lumps" of buy-in and cash-out at the two ends.



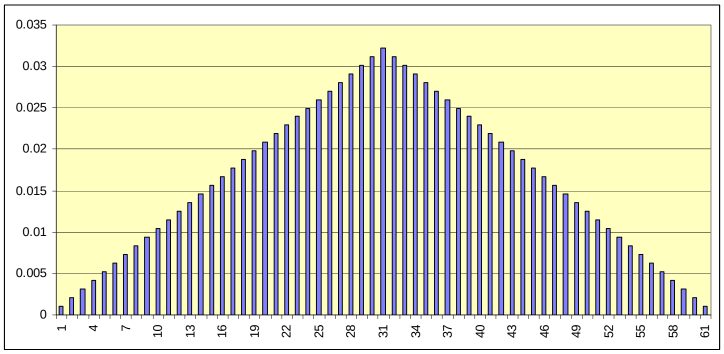


We can progress through an expanding spiral of statistical, trigonometric, and other mathematical functions, adding new ingredients to the triangular foundation until a higher minimum return has been achieved and verified. This process of interrogating or "pinging" the data is the mathematical sonar spoken of in this paper. This mixing and testing is done experimentally rather than theoretically (essentially by trial and error, adding a little of this and a little of that until an optimal mix is reached). The mathematical sonar builds its way through this ascending series of functions until a point of diminishing return is reached and further complications produce little or no improvement in minimum return, and run the risk of fitting or "chasing" the statistical "noise" in the data. Here is a sample procedure:

> Try a buy-and-hold function. If this doesn't work for p of reasonable length, discard it. If it does work, modify it with weights of the subsequent steps.
>
> Try a single moving average (flat purchase sequence and flat selling sequence). Find a good reasonable "p." Use this in place of, or as a supplement to, the buy-and-hold strategy. If buy-and-hold didn't work at all, use this as the new beginning.
>
> Try a double moving average triangular plan. Use a smoothing length suggested by Fourier spectral analysis or by an inspection of the peaks and troughs of the market price data series. If nothing else worked, start with this plan. If previous plans worked, use this to supplement the previous plans. Adjust the p and the s and the mix until no further improvement is apparently possible.
>
> At each new stage, introduce a new function into the mix: sine and cosine (and their negatives), bell-shaped curves, and so on.
>
> Also, with each new stage, experiment with truncations (chopping off the tails). The weight lost can be added evenly to the rest of the distribution, or it can be added entirely at the beginning and end of the distribution or spread out in the first few and last few periods of the distribution. Keep changing the numbers and the mix empirically (trial and error) until an optimal minimum return is reached (i.e. things don't get better by making new changes).
>
> When do you stop adding new functions to the mix, since this obviously can be continued indefinitely? When the minimum annualized return no longer increases significantly – in the preferred form, when the minimum return no longer rises by 0.10% (or some other threshold amount) by adding a new part to the mix.
>
> Another tool is a ***pure*** genetic-algorithm approach. Let all the values of f(t), the number of units of the index purchased every year or month, vary ***separately and arbitrarily,*** as long as the total weight under the curve is renormalized to 1. Those weights constitute the "genes" or "DNA" of the function. One at a time, adjust one of the weights up or down until an optimal minimum return is attained. Then adjust another weight, and another, seeking to further improve the minimum return. When no further improvement is apparent, stop. One danger of this process is that of chasing the noise or irregularities in the particular market data series m(t) rather than of capturing its systematic properties. It is therefore necessary to try only those combinations of weights that "make sense" or "look elegant," to use a mathematical phrase – in other words, that seem to follow a logical pattern or theme. It is also appropriate, though not absolutely necessary, to insist that the function be symmetric, so that its "right half" is the mirror image of its "left half," as is the case for a triangle. In such a case only one half of the distribution has true independence.

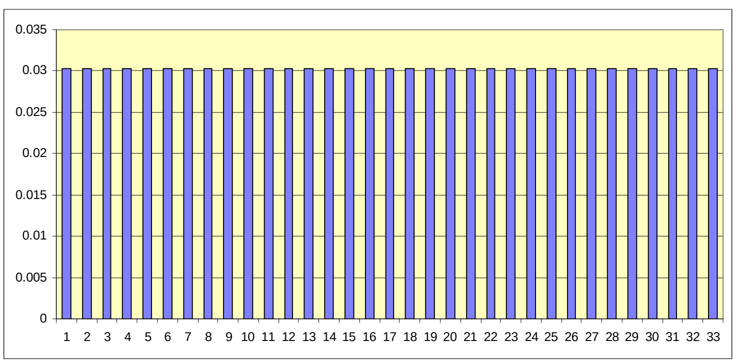
The reader is invited to consult the supporting Excel files for sample cases of how the returns develop and how a robust minimum return is experimentally verified. Sometimes I have used averages of different lengths, but always near to 30 years or so. Exact cycle length is not a requirement for this method.

In "File 1 FULL EXTENDED TRIANGLE DISTRIBUTION," the full double-smoothed triangle is used. [***A fuller and more technical explanation of the process, with detailed description of spreadsheet work, appears in the following section of this paper.***] The ***minimum*** annualized return is 1.41% (cell C1) and the ***median*** annualized return is 2.19% (cell E1). In this and all spreadsheets I have used ***s = 15 years*** for the time shift between the buying and selling distributions. This plan gives a very good long-term return, after inflation, and with dividends yet to be added in. The weighting plan, lasting 61 years, based on 31-year averages, is shown below. All the weights add up to 1.



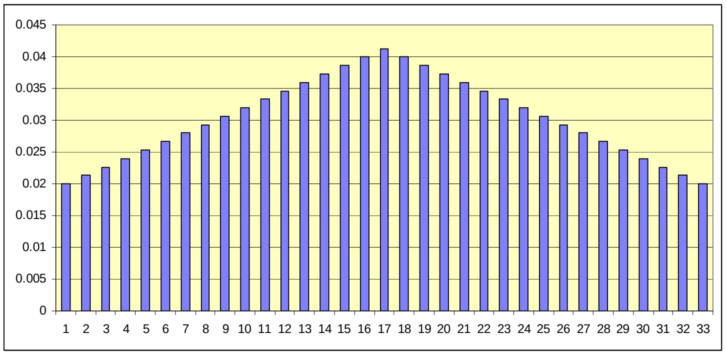
The supplementary file "File 1A FULL EXTENDED TRIANGLE DISTRIBUTION with graphs" includes the same investing plan, but also gives some of the graphs of the constant-dollar Dow and its moving averages.

The Excel file "File 2 FLAT DISTRIBUTION" gives a "flat" single 33-year moving-average plan.



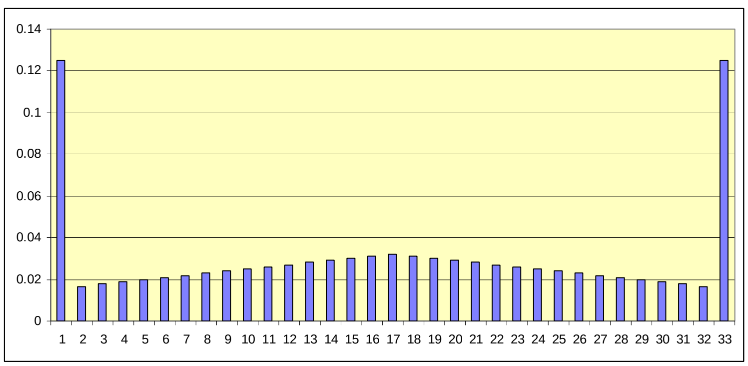
The minimum annualized return is only 0.57% and the median return is 1.66%, decidedly inferior to the double-smoothed triangular plan.

The Excel spreadsheet "File 3 TRIANGLE WEIGHT TRUNCATED DISTRIBUTION" represents an attempt to shorten the time needed to carry out the triangular plan of File 1 by chopping off parts of the triangle and redistributing the weight throughout the rest of the purchase and sales plan.

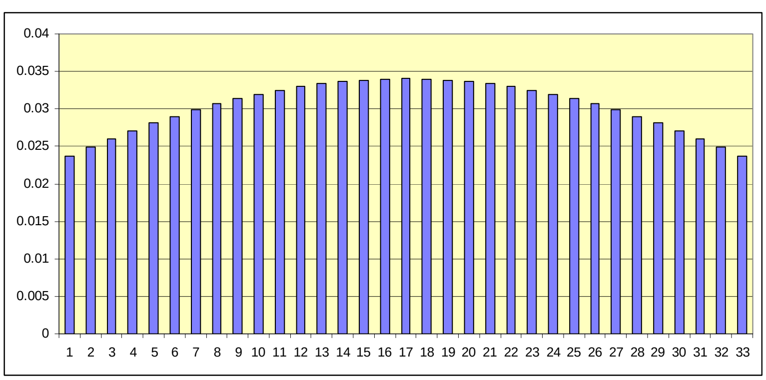


Unfortunately this has resulted in a ***negative*** minimum return of -0.21% and a median return of 1.46%. This has hurt the results rather than helped them.

To keep the total time span to a reasonable length (relative to human life) while not losing the tails so badly, we will look at "File 4 TRIANGLE WEIGHT CONCENTRATED DISTRIBUTION," which loads the cut-off tails entirely at the first and last years of the remaining parts of the plan. The total time needed for the buying (and also for the selling) plan is 33 years, and the minimum return has risen back to a positive 0.55% with a median return of 1.89%. This is better than the previous truncated plan.

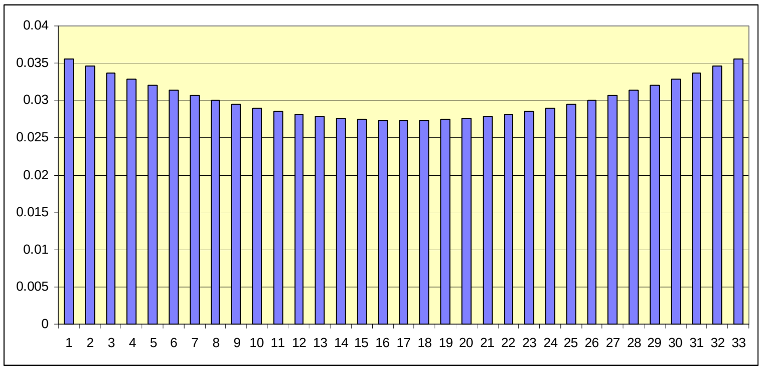


Next, "File 5 COSINE DISTRIBUTION" represents an attempt to try an adjusted cosine distribution on its own (not combining it with a triangle, which would be a more advanced step).



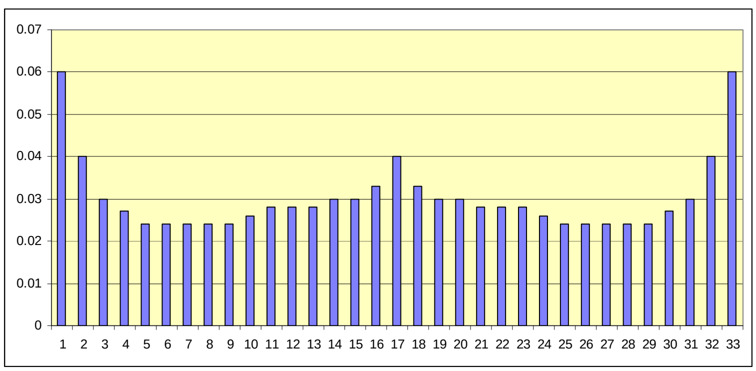
This plan has a disappointing minimum annualized return of 0.21% and a median return of 1.57%.

Let's try "File 6 INVERTED COSINE DISTRIBUTION," which is a combination of a flat buying pattern and an ***inverted*** cosine function.



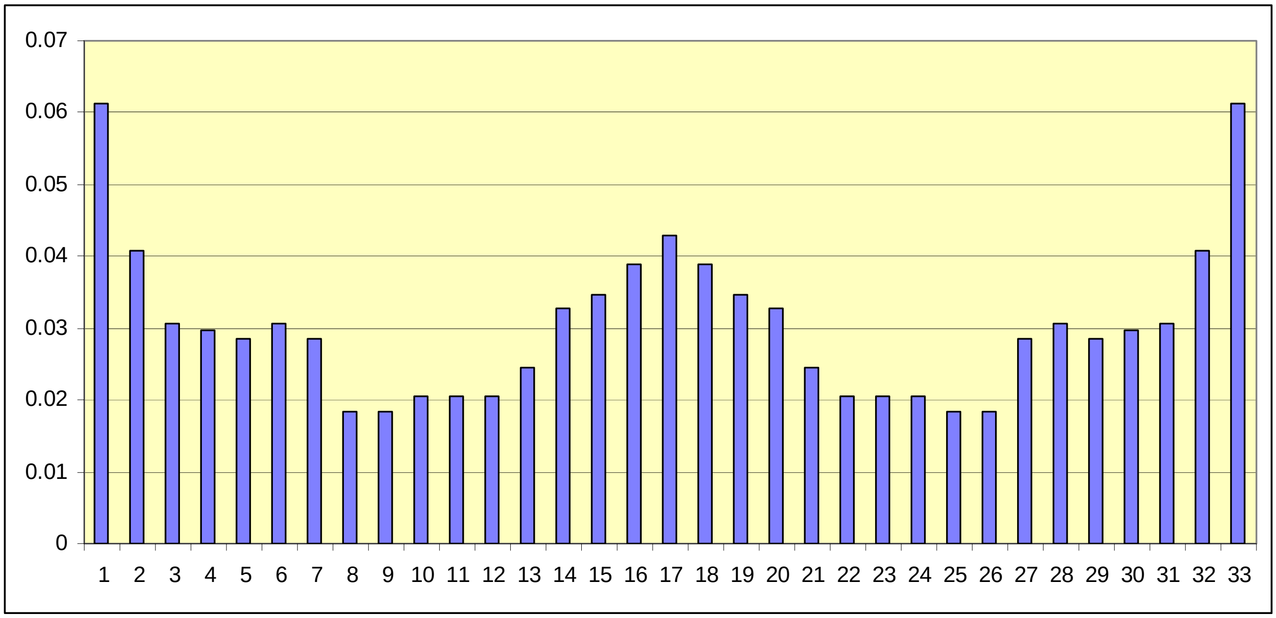
This plan has a better minimum annualized return of 0.69% and a median annual return of 1.70%.

Next, "File 7 EMPIRICAL SYMMETRIC DISTRIBUTION" is our first attempt at a full genetic-algorithm approach. The only requirement is that the weights be symmetric. By experimenting with the raw weights in column J, variations in the total distribution weights in column L can be tried. The actual distribution was built by starting with the insights that triangular schemes, concentrated triangle schemes, and inverted cosine schemes had their merits. After a genetic-algorithm process (that is, tinkering around) I got the distribution below.



The results have improved – to a minimum annual return of 0.715% and a median return of 1.77%.

Finally, "File 8 SECOND EMPIRICAL SYMMETRIC DISTRIBUTION" represents a further attempt at genetic-algorithm tinkering, allowing more variation into the individual weights that make up the "genes" of the buying and selling plans.



This is the best "shortened" plan so far. The minimum annualized return has risen to 0.824% at the cost of lowering the median annual return to 1.76%. But at least there are no "minuses" (losses) in the annualized return, and a reasonable expectation of 1.76% – plus dividends and after inflation – is quite good! For instance, allowing 3% as a historical rate for dividends, this is better than can be expected from Treasury bills, and the risk associated with the stock market has now been smoothed out. Actually, the return will be even ***higher*** because the dividend yield tends to be larger when the market is lower – precisely the best time to invest when future appreciation is likely to be greater. Conversely, the dividend yield tends to be lower when the market is high.

There is a danger in carrying the tinkering too far; it can find itself carefully fitting the irregular details of the data series rather than really following the essentials of the cyclical motion. It also looks like we are approaching a point of diminishing returns, with only modest increases in the minimum return.

None of the "shortened" plans have a minimum return anywhere near the 1.41% of the full-triangle plan, or a median return near the 2.19% of the full-triangle plan. That is, there ***are*** costs associated with shortening the plan, whatever tinkering is done to make things better. On the other hand, the clear benefit of shortening the plan is that the plan ***is*** shorter and takes a lot less time to carry out!

Clearly, the possibilities are almost endless. But the key breakthrough has been made: ***robust minimum return investing plans are possible!*** It ***is*** possible to robustly and reliably capture the long-term return of the market. Even with making some sacrifices to shorten the time of the plan, it ***is*** possible to get a good annualized return with a reasonable expectation of avoiding losses altogether.

INTEGRAL INVESTMENT MEASURES AND ROBUST RETURN: A TECHNICAL STUDY

This section of the paper goes into more detail, presenting a technical development of the mathematical "sonar" methodology to be used. Some of this material will be repetitive in that it overlaps the work of the less technical overview that precedes it. However, this section will proceed more deeply and provide a detailed and mathematical description of this key part of the present method and approach.

The major components of the mathematical "sonar" process in the present study are described in three words: ***functional, parametric, genetic.*** The process goes through the three stages (as appropriate) over and over until an integral measure solution is found which is believed optimal for the historical data series according to a criterion such as minimum return. We will define the three key words.

Functional. Mathematicians have long known that any function, under certain reasonable restrictions, can be described as the weighted sum of a series of component functions such as sine functions of different cycle lengths; an example is the Fourier series decomposition of a function. For an exact decomposition it may be necessary to resort to an infinite series of functions, but in practice a close representation is usually possible after using relatively few terms in the series. The functions must be mathematically linearly independent. Also, it is preferable, but not absolutely necessary, that the functions in the series be mathematically orthogonal to each other ("orthogonal" is a special mathematical property). In this study, orthogonality will not be required. Instead the functional process will use a series of independent (but not necessarily orthogonal) functions that ascends a conceptual ladder.

The functional operation is the highest level of the overall method of this study. It consists of beginning with the use of a single measure function, usually a very simple one. Parametric and genetic methods are used to find the best form of this single function that historically yielded an optimal solution when studied against the historic data series.

Once the possibilities of this function have been exhausted, a second measure function, linearly independent of the first, is added to the "functional set." Then, parametric and genetic methods are used again to find the best form and combination of functions that can be built from the two functions in the function set.

When it is believed that an optimal solution is attained, a third measure function is added to the function set. Parametric and genetic methods are then used to find the best form and combination of functions that can be built from the three functions in the function set.

This process continues until a point of diminishing returns is reached, where introducing additional functions yields little or no improvement in the return of the "best measure" built from the group of functions in the function set. In fact, at some point introducing additional functions is no longer appropriate, if it yields little or no improvement in return, and even that gain seems to arise from having "chased" the detailed noise of the data series rather than any systematic improvement in investment strategy; and if the conceptual complexity of the measure function substantially increases with little or no improvement in return or in our understanding of the workings of the market data series. In some cases it may be appropriate to drop one or more functions from the function set if it is clear that their inclusion made little contribution to the return.

Parametric. Within each stage of building the functional set, parametric variations are experimented with, using the existing historical data series, in search of the best return (in the preferred form, as measured by minimum return) possible with the functional set currently in use. Mathematically, a "parameter" is a number or other characteristic that describes and defines variants of a family of functions. Examples of parameters include: the width of a flat or triangular distribution; the cycle length of a sine or other trigonometric function; or the relative weightings of a weighted sum of two or more separate functions. In the preferred form, the optimum parameter or parameters are arrived at through empirical experimentation rather than through a closed-form approach using mathematical equations. Once optimal values are found and the possibilities of parametric variation are exhausted, the method returns to the functional approach by enlarging the functional set to include another function.

Genetic. In the present development, in the preferred form, this methodology is not used until the functional and parametric methodologies have reached an appearance of "maxing out" with few additional gains apparent or envisioned. In the genetic approach, the yearly or monthly investment amounts (the values of the investment measure function) are considered as separate and independent "DNA units" or "genes" of the total distribution. In the preferred form, these individual weightings are adjusted in small increments up or down one at a time, to see if the desired form of return (such as minimum or median return) increases or decreases. When a maximum return is reached, a second "gene" is adjusted, and so on, until an optimal "genetic sequence" of weightings is obtained such that any further adjustments of any of the genetic weights produces a decrease in return rather than an increase. This genetic-algorithm methodology results in the measure function "hill-climbing" its way through the space of possible functions. This will usually, but not always, arrive at an optimal solution for the function set currently in use.

It is possible that the genetic method will arrive at a function which is a "local maximum" or "local hilltop" in measure function space. That is, small adjustments to the measure function in any direction no longer produce gains in the return, however that is measured. But it may yet be possible that there exists a more distant and higher "hilltop" in measure function space with a higher return. Since the existing set of functions, parameters, and genes has been exhausted in its possibilities, at this point the analyst may make large changes in the functions, parameters and genes, or may introduce yet another function to the function set and begin parametric and genetic adjustments all over again.

Genetic methodology may improve the total return considerably, or it may produce only a small improvement in return at the cost of greater complexity – in such cases because it has only followed, matched or "chased down" the detailed noise in the data series rather than having provided a systemic leap in understanding and yield.

We will now proceed to illustrate the methodologies of this study using the yearly Dow index as the basic data series.

The simplest ***function*** to use is that of a single unit of purchase followed some time later by a single sale. This is the classic ***buy-and-hold*** strategy. There is only one ***parameter*** to be considered at this stage – namely the number of years the purchase is to be held until it is sold.

The supporting file "File A15 buy and hold 15 years" presents the results for initial purchase years from 1900 to 1989 and a parametric hold time of 15 years.

> In that supporting file, column A gives the year and column B the Dow index on a yearly basis.
>
> Column C gives an inflation index which is multiplicatively adjusted to generate a Deflator in Column D.
>
> Dividing the Dow by the Deflator gives the Real Dow in Column E.
>
> Allowing a time span of 15 years gives the value of the Real (constant dollar) Dow fifteen years later, presented in Column F.
>
> Dividing Column G by Column F suggests a fifteen-year multiplicative return.
>
> This return is in turn converted into an annualized return using natural logarithms and dividing by 15; the results are displayed in Column H.
>
> The "weighting" system is very simple here, as shown in Column I; it consists of a single purchase and nothing more.
>
> The sum of the weights at the top of Column J is easily seen to be 1.
>
> So when the weights are "normalized" to make their total 1 in Column K, no changes need to be made. The actual distribution function is rather uninteresting, but it is graphed in the spreadsheet.
>
> The minimum annualized return appears in cell K39, and is repeated in cell C1. It is -6.957%, the unhappy result of buying in 1967 and selling in 1982, losing almost two-thirds of the initial investment in constant dollars.

The median annualized return is 1.923%, which appears in cell K41 and is repeated in cell E1.

The mean annualized return is 2.141%, appearing in cell K43.

The standard deviation of the annualized returns is 4.474%, shown in cell K45.

The median overall return after inflation and without dividends is 1.923%, which may appear attractive, but this strategy is not ***robust*** because the minimum return is -6.957%, indicating that it is possible to lose money if one buys at the wrong time – buying in 1967 and selling in 1982.

At this point we continue the ***parametric*** analysis and look at a 30 year holding time. The supporting Excel file “File A30 buy and hold 30 years” presents the results for initial purchase years from 1900 to 1974 and a hold time of 30 years. The minimum annualized return is -0.787% and the median return is 1.445%.

Next we will continue the ***parametric*** analysis and raise the holding time to 45 years. The file “File A45 buy and hold 45 years” presents the results for initial purchase years from 1900 through 1959 and a hold time of 45 years. The median return is 2.152% but the minimum return rises only to -0.492%. Buy and hold is not a good robust-return strategy! In a separate paper, the author showed that it ***was*** possible to get a positive robust minimum annualized return (after inflation, without dividends) from between 0.5% to 1.0% if the parametric hold time was raised to ***70 years,*** but this is not practical for real-world investors.

We will now return to the ***functional*** approach and add a new function: a flat distribution of buying over a certain number of years, followed by the same distribution of sales with a time lag. There are two ***parameters*** here: the number of years over which the buying (and selling) is spread out, and the time lag in years between the start of buying and the start of selling.

The supporting Excel spreadsheet “File B1 flat distribution 33 years wide lag 15 years” derives the returns with the parameters “33” and “15” for different years when the strategy begins.

Note that the raw, flat weights appear in column J, all equal to 1.

Their sum, which is 33, is at the head of column K.

Then, to make the total area under the distribution equal to 1, the appropriate division is carried out and the distribution weights appear in column L, all equal to approximately 0.0303.

Columns M through CF represent the same weighting arrangement, but set forward one year at a time until in column CF the weighting plan terminates in the year 2004.

Column CL represents the product of the cells in column F and column L. Here the weightings of column L are each multiplied by their yearly constant dollar “real” Dow levels in column F. The ***sum total*** of that column in cell CL111 is the total money expended in ***constant dollars*** to buy that weighted distribution of the Dow across the years 1900 to 1932.

Columns CM through FF represent the product of the cells in column F (which is always fixed) and columns M through CF respectively (which shifts one year at a time). The sum of these columns, in cells CM111 through FF111, shows the total money involved in constant dollars associated with the weighted distribution of the Dow, across the appropriate time span. The rightmost column is column FF, which shows the total money or “value” of the Dow in constant dollars when bought (or sold) starting in 1972 and ending in 2004, with a flat weighting plan.

Now, the plan calls for an investment function followed by the same selling function with a time lag. In this particular example the time lag is 15 years.

The amount of money received, in constant dollars, across the years when the investments are sold with a 15-year time lag, is precisely the sum of the corresponding column moved 15 years to the right. ***That numerical sum represents the appropriate total sum of constant dollars involved, whether expended in investment buying or received through the program of selling!***

If an investor starts in 1900 and ends in 1932, the constant-dollar money expended to buy these Dow units purchased from 1900 to 1932 is 675.75, in cell

> CL111. These units of the Dow are sold in exact correspondence fifteen years after they are bought, from 1915 to 1947. The constant-dollar money obtained from the sale is found in column DA and the total sum is 797.90, displayed in cell DA111.
>
> Another way of lining up these fifteen-year shifts is shown in cells CL113 through EQ113, which is the “111” row shifted fifteen years right.
>
> The net return over fifteen years is in cells CL115 through EQ115, which is cells CL113 to EQ113 divided by CL111 to EQ111 respectively.
>
> The annualized yearly return is found by taking the logarithms of the “115” row and dividing by 15. This is displayed in cells CL117 to EQ117.
>
> Cells CL119, CL121, CL123, and CL125 represent the minimum, median, mean, and standard deviation of the total set of annualized returns. For reading convenience these numbers are copied into cells K39, K41, K43, and K45. The minimum and median annual returns are again copied into cells C1 and E1 respectively.

The minimum annualized return is 0.574% and the median return is 1.664%. This is decidedly better than what we saw for the buy-and-hold strategy since at least the minimum return is positive. This investment measure function may ***now*** be said to have a ***robust return.***

Let’s do some ***parametric*** analysis and increase the time lag to 30 years. The results appear in the Excel spreadsheet “File B2 flat distribution 33 years wide lag 30 years.” Because of the extended number of years necessary to carry out such a plan, it can’t be computed for all of the years involved, but the spreadsheet gives information based on decades of tests.

> The development of the spreadsheet is very similar to that of the previous file. The only difference is that cells CL113 to EB113 represent the “111” row shifted forward by thirty years instead of fifteen years. Row 115 is computed the same way as before, and in row 117 the annualized return is computed by dividing the appropriate natural logarithm by thirty instead of by fifteen. The cells displaying the minimum, median, mean, and standard deviation of the annualized return play the same roles as in the previous file.

The minimum return has risen to 1.222% and the median return to 2.058%. The plan has improved in its return, but at the serious cost of taking much longer to actually carry out. In fact, this plan may require an unacceptably large number of years to bring to a conclusion. This sort of trade-off between minimum return and total time necessary is frequently observed.

Let’s change the ***parameters*** and decrease the time lag to 15 years, and also decrease the total investment time span to 15 years. The result is in “File B3 flat distribution 15 years wide lag 15 years.” The easiest way to do this is to start with File B1 and reduce all weights to zero after the fifteenth year.

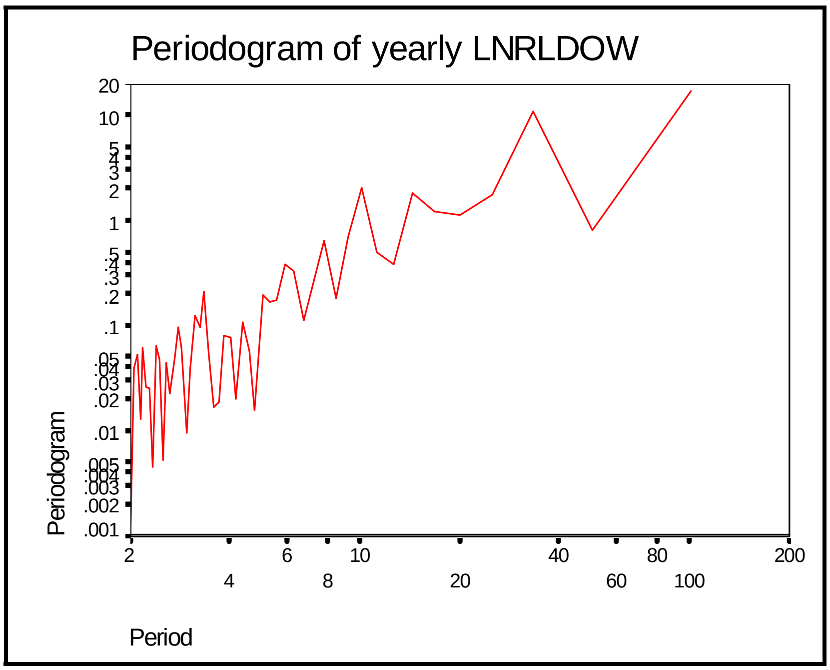
> In this file, the weights begin in cell J4 and extend only to J18, a total of fifteen years. The weights below J18 have been set to zero.
>
> These weights add up to 15 (cell K3) and the normalized weights are adjusted to 0.06667 each, shown in cells L4 to L18. The columns to the right, up to column CX, are shifted forward by a single year per column as in previous files.
>
> Columns DE through GQ are the same as in “File B1” except that many of the entries are zero because their weights have been set to zero. There are more columns than in “File B1” because the shortening of the plan means we must consider more possibilities; the weights can begin as late as 1990, ending in 2004.
>
> The sums are again displayed in the “111” row, from cell DE111 to GQ111.
>
> These sums as shifted fifteen years forward appear from cell DE113 to GB113.
>
> Row 115 represents row 113 divided by row 111, and then the annualized return is in row 117, after dividing the appropriate natural logarithm by 15.

The cells displaying the minimum, median, mean and standard deviation of the annualized return play the same roles as in earlier files.

This plan takes far less time to carry out (15 years of buying, followed by 15 years of selling) but is in practice unacceptable, because it has a minimum annualized return of -3.747%. The shortened time span has introduced a large degree of risk. The plan is not robust.

Note that I have preferred to work in "constant dollars" almost as though this constituted a currency of its own. This is legitimate and appropriate because we are interested in the true annualized return after inflation, not merely in current-dollar inflated numbers.

Now we return to the ***functional*** methodology and introduce the possibility of double-smoothed moving averages and consequent triangular measure functions. Let's look more deeply at why a 30-year or 33-year time span was used in the first place. We have looked at the Dow index after inflation. But next we take the natural logarithm of that series, called "LNRLDOW," so that a doubling from 1000 to 2000 will look the same as a doubling from 2000 to 4000, and so on. A spectral analysis from an earlier study of ***that*** series generated the following periodogram:



Interpreting the periodogram above, there is a "high point" or "resonance" on the graph between a 30-year and 40-year period, suggesting the existence of a reasonable homeostatic cycle of about that length, and suggesting a smoothing (moving average) or double-smoothing (double moving average with triangular measure function) using such a time span. ***This*** is where the whole idea of using "30 years" or "33 years" came from in the first place!

Motivated by what was seen with the use of double-smoothed moving averages, we look at "File C1 full triangle lag 15 years." This is the same as the "FILE 1 FULL EXTENDED TRIANGLE DISTRIBUTION" referred to in an earlier section of this paper, but I am trying to keep things neat and organized. Several of the files in this section will be the same or almost the same as files referenced earlier; but again, I have tried to keep each part of the development independent and be very painstaking in exhibiting the work step by step.

In this "File C1," the Dow after inflation appears in column F and the Dow fifteen years into the future is shown in column L.

The triangular weights extend over more than sixty years, from cell O4 to cell O64.

The total weights add up to 961 (cell P3), and they are normalized by dividing by this number in cells Q4 to Q64, so that they add up to 1. In other words, one "unit" of the Dow is purchased over the total span of this plan, to be sold with a 15 year time lag.

Columns R through BI represent the weights of column Q, shifted forward one year at a time. The last column, column BI, begins in 1944 and ends in 2004.

Column BO is the product of the constant dollar Dow in column F, multiplied by the weights in column Q. It represents the constant dollar value of the purchases (or sales) of the triangle beginning in 1900 and ending in 1960.

Columns BP through DG are the product of column F (which does not change) with columns R through BI respectively (moving one year forward, one column to the right, each time). The rightmost column (BI) represents the constant dollars associated with the purchase (or sale) of the Dow according to the triangular distribution, starting in 1944 and ending in 2004.

As before, the "111" row, from cell BO111 to cell DG111, represents the total sum of constant dollars involved in a triangular plan.

Row 113, from cell BO113 to CR113, follows the reasoning of earlier files and represents the sum total of the 15-year-lagged triangle – that is, the total constant dollars received when the lagged triangle is sold.

Row 115 represents the return over 15 years and row 117 shows the annualized return by dividing the appropriate natural logarithm by 15.

Cell BO119 is the minimum of the annualized returns from BO117 to CR117.

Cells BO121, BO123, and BO125 show the median, mean, and standard deviation of the annualized returns.

For convenience, cells BO119 to BO125 are copied into cells P39 to P45.

The minimum and median returns are also copied into cells C1 and E1 respectively.

The minimum return is 1.412% and the median return is 2.188%. This looks quite good, but it will take about 76 years to carry out this plan. The triangular weights are in column O, normalized into column Q and so on. There are 61 years of buying overlapped by 61 years of selling (with a 15 year time lag in the triangle). The robust minimum return looks good, but the time necessary may be unacceptably long. Increasing the time lag to 20 or 30 years might increase the minimum return but would also increase the time needed to fully carry out the plan even further.

Let's change the parameters and look at "File C2 short triangle lag 15 years." The investment span is only 15 years long and the time lag is 15 years, so the plan needs only 30 years to complete. But the minimum annualized return is -4.723%, which is unacceptable even though the median annualized return is 1.989%. The shortening of the plan has made it more unstable. This plan is not robust!

Note that there are more columns in this spreadsheet since we must consider triangles beginning as late as 1990 (ending in 2004). I did not extend the analysis through 2007 because most of the work was done earlier, because in some cases there was not enough room in Excel, and because of personal time constraints. However, extending the work by three more years will affirm, not refute, the main conclusions of this paper.

Let's look at an intermediate parameter level in "File C3 middle triangle lag 15 years." The investment span is 33 years followed by 33 years of sales (lagged 15 years, for a total completion time of 48 years). This might be acceptable in the light of a working-lifetime-plus-retirement of 50 or 60 years. But the minimum annualized return is still negative, at -1.685%, as shown in cell CL119.

Before going on in the study, I would like to say that the Excel spreadsheet format is ideally suited to the analysis in this paper. It is easy to modify or "tweak" a weighting distribution and to adjust the time lag. All the cells are recalculated automatically and appropriately.

I have left the formulas and calculations in the supporting Excel files "live" rather than presenting them as "dead numbers." People will be able to experiment to their hearts' content with alternative weightings and approaches. This will hopefully represent and encourage the innovative spirit which characterizes our country at its best.

Next, look at the ***truncated triangle*** in "File D1 truncated triangle." [This is the same as "FILE 3" in the earlier section of this paper.] This plan has 33 years of investment and 33 years of sales, time-lagged by 15 years. The minimum return is still negative, at -0.208%, but not as bad as some plans previously studied.

One might think that the use of a truncated triangle represents a return to the ***functional*** level of analysis, but this is not the case. In fact, the truncated triangle is simply a linear combination of a flat distribution and a true triangle, so this is actually a ***parametric weighting*** of the two measure functions! In "File D2 truncated triangle" we see that the measure function of this plan is actually 14 times a flat distribution plus 1 times a triangular distribution, all normalized to have a total weight of 1; see particularly columns J and K of that file, with the weightings in cells J2 and K2.

We can now vary the weighting parameters. In "File D3 truncated triangle" we increase the weight of the flat distribution to 20, increasing the minimum return to -0.027%. In "File D4 truncated triangle" the flat distribution has its weight increased to 30. The minimum return increased to 0.138%, finally entering positive territory. In "File D5 truncated triangle" we put a ***negative*** weight (of -1) on the triangular distribution, putting greater weight on the ends than in the middle. The minimum return increases to 0.448%, suggesting that move was a good idea. But this still was not as good as the minimum return of 0.574% from a flat distribution as described in "File B1."

In "File D6 truncated triangle" that negative weight reaches -1.5 instead of -1, but the minimum return becomes negative. Instead, start with a weight of -1 for the triangle and change it to -0.9, then -0.8, and so on. The minimum returns get better and better until at a weight for the triangle of -0.3, a minimum return of 0.716% is attained in "File D7 truncated triangle." This is superior to the minimum return of the flat distribution, indicating that true improvement has been made. Here the -0.3 represents an optimum obtained from that parametric style of variation, since changing this number to -0.2 and to -0.4 both result in an inferior minimum return.

Note that we have taken advantage of the spreadsheet software – it is easy to vary the parameters and undertake all sorts of experiments!

Still other parametric variations are possible. We could take the distribution from File D7 and put in a single purchase at any point (introducing a weight from the original buy-and-hold function). The parameters here would be the point of introduction of the single purchase within the time span of buying/selling, and its weight or amount in the total distribution. One variation on this is to put in a single large purchase at both the beginning and the end of the distribution, as appears in "File E1 truncated triangle with weights at end." Here we started with File D7 and introduced two weights at each end, of size 1. This involves adding a new column (J) to include these special actions, with its own function weight in cell J2. The minimum return increased slightly, to 0.723%. In contrast, ***taking away*** two weights at each end, of size 1, in "File E2 truncated triangle with weights at end," using a "weight" of -1 for the end purchases, decreased the minimum return to 0.709%. That looks like the wrong direction to go.

Thus, we will increase the weight of the "end purchases" to 2, 3, and so on, for as long as the situation improves. But it does not help: "File E3 truncated triangle with weights at end" uses a weight of 3 at each and the minimum return is only 0.714%. Upon experimentation it is found that a weight of 1 is best (the minimum return is inferior for weights of 0.9 and 1.1).

We have taken advantage of the spreadsheet's capabilities. The parametric approach has room to try a lot of things – but it seems to have about exhausted its possibilities for now. We could keep adjusting the three weights, but let's try something else.

We could return to the ***functional*** approach and add more functions to the mix, such as cosine functions (or their negatives) of varying lengths and sizes, or shorter flat periods or shorter triangles (of only a few years' length) as "finishing touches" or "perturbations" to the distribution.

However, let us immediately proceed to the full ***genetic*** approach of varying the weights arbitrarily. In "File F1 empirical symmetric distribution, genetic" we see a possible distribution. If we require the distribution to be symmetric, it is entirely determined by cells J4 to J20. This distribution has a minimum annualized return of 0.715% which is not quite as good as some numbers we have already seen.

The ***genetic*** approach means varying the individual cell weights experimentally to see if the minimum return increases. This carries out a hill-climbing genetic algorithm in "function space," the set of possible functions. No explicit formula for a function (such as a triangular or a cosine function) is needed since we are choosing the yearly weights individually.

Again, note that this methodology definitely exploits the features of a spreadsheet!

We find that "File F2 empirical symmetric distribution, genetic" is a little better. It is obtained by trying to get the "best" weighting in cell J4 (representing the first, and also the last, year of purchasing); then, having done that, trying to get the best weighting in cell J5, and so on up to cell J20. The minimum return has increased to 0.867%.

From this point on we can return to adjusting cells J19 up to J4, until a "hilltop" function is reached where none of the cells need be adjusted further; and adjusting any of the "genes" up or down by 1 only serves to decrease the minimum return. This would be a local "hilltop maximum" in function space, in the space of investment measure functions of the specified extent (with the requirement of symmetry).

However, the full genetic approach has some aspects which suggest caution. First, it may be finding and fitting specific weights which are well suited to the particular details of the historic data series, but are not likely to produce such a high performance in the future. In other words, the weights have "chased the noise" in the data series rather than added any conceptual understanding. In contrast, with distributions such as the triangular plan we could have more confidence of understanding what was going on and why we had good reason to systematically expect a robust minimum return.

Second, the genetic approach may make lots of adjustments for only a marginal gain. At some point a level of diminishing return is reached where little net gain is achieved, and there is loss in understanding and in the solidity of future expectation. If we can't understand why something works, we can't be so confident that it will reliably work in the future! In such a case we may want to consider the analysis complete, or return to the functional level or change the parameters (such as widening the span of the investment measure function).

Furthermore, the genetic-based investment measure functions developed here are not suited to personal demographics. Most people are able to make only small investments at the beginning of their careers (typically, in their twenties), while the period of greatest saving and investment is in middle life (typically, in the forties and fifties). The period of selling (or at least, the dominance of selling over buying) will probably begin in a person's sixties – and yet the plan should make provision for today's long life spans and a possible long retirement extending until eighty-five or even ninety years of age.

The "full triangle" investment function is better suited to the actual economics of individual careers and savings patterns, but the example hitherto given takes too long (seventy-six years) to carry through. Truncations of the full triangle did not give a good robust minimum return.

To investigate further, we return to the parametric approach. We shall see that the minimum return increases considerably with modest increases in the ***width*** of the truncated triangle and also with the ***time lag*** between the buying and selling distributions. Both of these increases add to stability.

In the supporting Excel file "File G1 truncated 45 years wide lag 20 years," the minimum annualized return is a more attractive 1.117% and the median annualized return is 1.956%, for distributions with starting years from 1908 to 1950 and ending years from 1952 to 2004. [This file has also been saved as "FILE 9."] Both of these numbers are better than the returns from the shorter plans examined here, even when the shorter plans used genetic adjustments. The addition of a few years to the width of the truncated triangle and the time lag between buying and selling has made for a better robust return. Note that the "117" row involves a division by 20 instead of a division by 15 to calculate the annualized return.

With this in mind, see the supporting Excel file "File G2 truncated 40 years wide lag 25 years." [This has also been saved as "FILE 10."] Here the minimum annualized return is 1.180% and the median annualized return is 1.966%, for distributions with starting years from 1910 to 1964 and ending years from 1950 to 2004. Note that the "117" row now involves a division by 25 to compute the annualized return. In the author's opinion, this is the best investment measure function investigated up to this point.

Let's look at this plan from the point of view of personal demographics. A person begins this plan (or the plan is begun on behalf of the person) with a low level of investment at the beginning of his or her working career, say at 25 years of age. The investment or buying period is most intense at 45 years of age and ends at age 65.

The selling plan is lagged by 25 years from the buying plan. Selling begins at 50 years of age and is most intense at 70 years of age; it terminates when the person is 90 years old. The balance shifts from buying to selling when the person is 57½ years old, moving toward retirement (early or otherwise). That is when the buying and selling functions cross, with the buying triangle moving down (lessening) and the selling triangle moving up (becoming greater or more important). As an aside, money released from selling, if not consumed by the person, can be invested conservatively, perhaps in Treasury bills (indexed for inflation or not) or comparable conservative investments, until it is needed.

Basically, I like this plan! It has a robust minimum return and a good median return of about 2 percent annualized, after inflation and without dividends. The breakthrough envisioned has been made.

The methodology has been explained. Several examples of investment measure functions have been given that historically obtained, and have expectation of obtaining in the future, a robust minimum return. These investment measures are themselves novel and non-obvious.

In the preferred form, genetic analysis is usually deferred until functional and parametric methods have been carried through several stages; then the genetic methodology is used to introduce modifications (mathematically, perturbations) to the measure function generated by functional and parametric methods. In alternative versions, genetic methods may be used throughout the process, or may replace functional and parametric methods entirely.

In the preferred form, minimum overall return is used as the measure of the effectiveness of an investment measure function. In alternative versions, other criteria may be used, such as mean or median overall return, or some entirely different criterion.

In the preferred form, only symmetric investment measure functions are considered. These functions are mirror-symmetric about their center, so that their "left half" is a mirror image of their "right half." In alternative versions the requirement of symmetry may be dropped so that a broader family of measure functions, symmetric or not, may be used.

In the preferred form, the distribution of sales is exactly the same as the distribution of purchases, except that it is time-lagged by a fixed number of years or months into the future. In alternative forms, this requirement may be dropped so that the distribution of sales may be different in form from that of investment purchases. The analyst will need to judiciously balance any improvement in return that results from widening these requirements with the issue of possible decreases in robustness – where any additional return arises from the peculiarities of the individual numbers rather than the deep fundamental properties of the index studied – and hence this additional return may not be so supportable as a future expectation.

In the example given in this paper, the Dow Jones Industrial Average, measured on a yearly basis, studied before or after inflation, is the fundamental data series used to measure the returns of investment measure functions. Alternatively, other "major market" data series may be used, involving stocks, bonds, commodity prices, national income, demographic numbers, real estate prices, or other data series – including "synthetic" data series such as price to earnings or price to dividend ratios. These series may be measured on a yearly, quarterly, monthly basis, or in some other way.

OBSERVATIONS AND REMARKS

First, some will wish to consider the topic of ***survivorship bias.*** Occasionally stocks are removed from the set of 30 that compose the Dow Jones Industrial Average, and others are included. This does not introduce a major unrealistic positive distortion to the returns envisioned in this paper. The weighting of the index is adjusted to avoid discontinuities associated with the removal and addition of stocks to the index. Also, recently it has become possible to actually buy and sell the index itself. In any case investors could sell stocks that are about to be deleted and buy those about to be included, without serious damage to the overall return. Finally, many indices such as the Standard & Poor 500, the Russell index, and the Wilshire index are built around far more than thirty stocks.

Second, consideration needs to be given to any inexact match between an investor's actual flow of available funds (and needs for withdrawals later) and the investments and sales called for in these integral plans. It may be that an investor has more money to invest than the plan calls for in its early years, or has lesser needs than the planned sales and disbursements in later years. It is therefore necessary to make conservative alternate plans for what to do with unallocated money. In one's early years, it might be wise to save for the down payment on a primary residence. In the later years, money could be put towards the college expenses of one's offspring or alternately held in a conservative form such as Treasury bonds until it was needed. In general, some conservative alternate plan should be derived for any unallocated money; on an overall basis the total rate of return from such an alternate plan combined with a good integral plan will still be good.

What shall be done if the investor has ***less*** money to invest than the integral plan calls for – and thus cannot purchase the number of "Dow units" that the plan specifies? Investors and firms should design ***modified approximations*** to the stronger integral plans – which allow for the possibility of reduced purchases for certain parts of their duration. It is agreed that the overall return will likely be less than for the best integral plans, but even a modified or mathematically "perturbed" integral plan can still yield a good and robust annualized return.

Third, it must be remembered that the major achievement of the present paper is in the ***robustness and consistency*** of the integral investment plans. These plans do not promise miracles. They do not purport to deliver 50% gains year after year through some particular trading arrangement. In fact, they do not apply to short-term trading at all. Instead, their merit is that they extract, over a period of decades, the long-term rate of return that was ***already there*** – but do so in a way that straddles the positive and negative parts of the business cycle and the irregularities of its motions. This paper offers no scheme or trick to generate incredible profits. Rather, the improvement is in increased ***stability and robustness,*** which is of considerable value for long-term and retirement-oriented investment.

NAVIGATION AND FORECASTING: REVIEW OF THE PHASE METHOD

The mathematical sonar developed in this research has also made possible, even in its elementary stages, the construction of navigational and forecasting tools through what is called the ***Phase Method.*** This author has written several published and unpublished articles and other material on the Phase Method.

Furthermore, after presenting the Phase Method and its robust navigational and forecasting techniques, a new and unpublished application will be developed in which the Phase Method is used to improve integral measure investing patterns.

The ***Phase Method*** converts trended homeostatic motion into an approximately circular form, which makes it very useful in navigating one's way through the business cycle. ***In contrast to integral investment measures, which straddle long homeostatic cycles to capture the long-term trend, the Phase Method removes the long-term trend itself to capture the movements of the long homeostatic cycle, while still smoothing out shorter-term motions.*** While mathematically akin to the work on integral investment measures, the Phase Method works in an opposite direction from that approach.

In summary, the Phase Method begins with a major market price series such as that taken from the New York stock market or a large real estate market (though these methods can be applied to any appropriate major market). This data series may or may not be adjusted to remove the effects of inflation. Often the natural logarithm of the prices (corrected for inflation or not) is taken to make the growth rate approximately linear, so that a price doubling from 1000 to 2000 looks the same as a second doubling from 2000 to 4000.

The long-term growth trend will be approximately linear and may be estimated by regression – or supplied as an estimated linear trend by a user. This trend may also be estimated using the last two previous peaks, troughs, or middle points. After subtracting the linear trend away, the remaining homeostatic motion is then smoothed, typically by applying a single moving average or a double-smoothed moving average, although sinusoidal and other functions may also be used. Another, and often preferred, way of doing this part of the work is to first do the smoothing or double-smoothing, and then estimate the linear trend, finally subtracting the trend away. The time span of the smoothing is shorter than what is typical of the large homeostatic cycle; here the intention is to smooth over short-term and medium-term motions, in order to ***retain and recover the main cycle*** in a reasonably smooth manner.

For the New York stock market, it is well known that there is a very approximate homeostatic motion of 40 months to 4 years in length. This may be related to our national election cycle or may arise due to internal macroeconomic forces – but whatever its origins, this approximate motion can be shown to exist through Fourier spectral analysis with a resonance of roughly four years or somewhat less. The construction of the smoothing techniques may be done using the same mathematical sonar as used in building integral investment measures, working in an expanding spiral through a variety of mathematical functions and operators until a reasonable and robust method is found.

The Phase Method then looks at the displacement of this smoothed motion over or under the long-term trend. This displacement of the (smoothed) market above or below trend will constitute the horizontal x-axis of the ***Phase Clock.***

In the same way, the ***rate of change*** or ***velocity*** of the market (rate of motion relative to the long-term growth rate) is estimated and smoothed as needed. This becomes the vertical y-axis of the Phase Clock.

Then, the position of the (smoothed) market is plotted as a moving dot tracing out an approximate circle. In this way, the Phase Method converts homeostatic cyclic motion into a more literal circular form.

This approach is a good ***real world*** technique. It can handle irregular and time-lagged data. It does not use or require ***deterministic*** knowledge (exact scientific equations of motion) of how prices move. It provides a general estimate of market position and likely future motions, and does so even if the trend line is estimated with a modest error or if other data is imperfectly or incompletely known. In this sense, it can be said that the Phase Method is ***robust.***

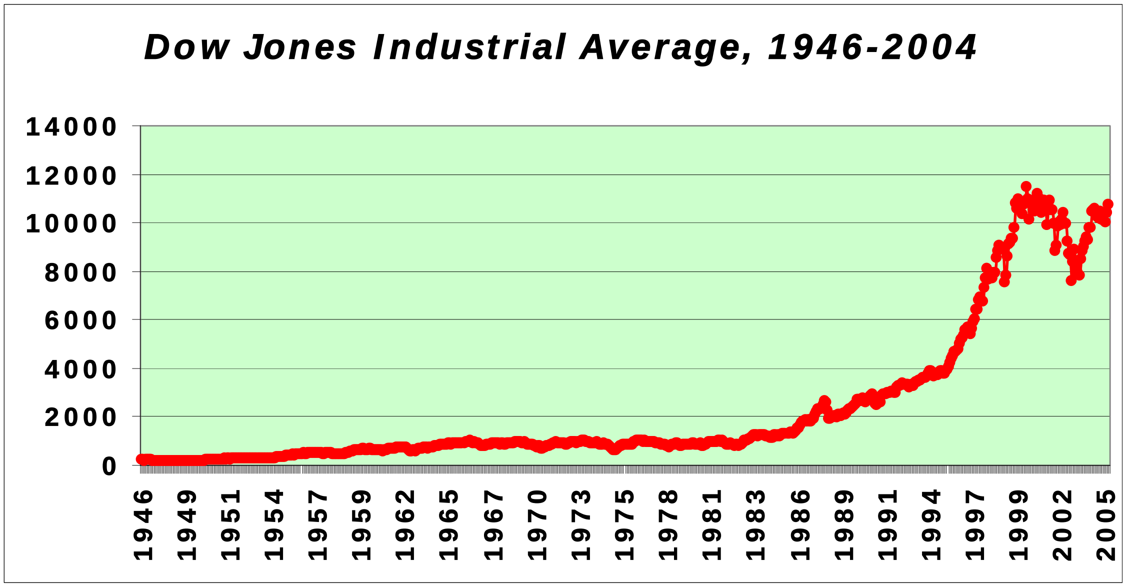
Throughout this paper I have been using the word “robust.” It is appropriate to give some informal definitions of that term.

The Phase Method and the methods of the present paper are robust in that they hold up reasonably well in real-world situations with irregular, imprecise, and time-lagged information. Informally, these methods can take a beating and still work well. Informally speaking, these methods are ***robust*** in the sense that they still work reasonably well even if they are somewhat wrong. Speaking casually, one can have things work out right even if they go wrong!

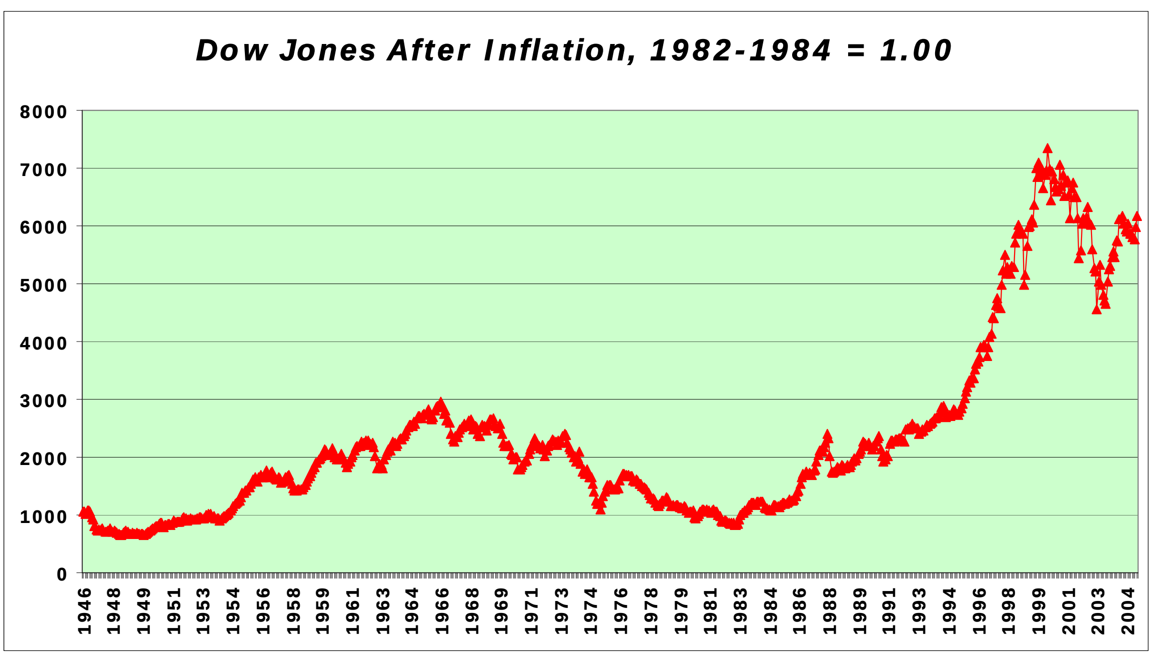
As we shall see, the Phase Method and its “Phase Clock” do not track the daily or monthly motions of the market, since those have been smoothed out. Instead, they track the idealized and smoothed motions of the market, which in most cases is what people with a medium-term or long-term time view would like to know anyway.

REVIEW OF THE PHASE METHOD AS APPLIED TO THE DOW JONES INDEX

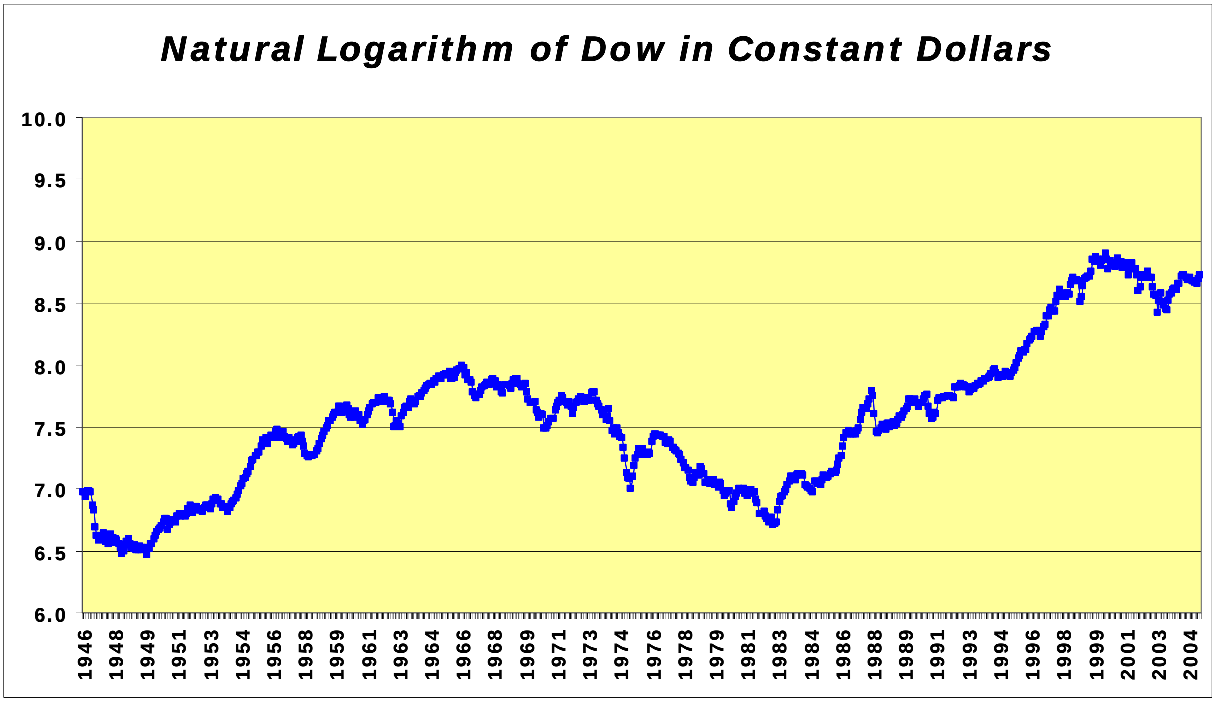
Let's look at some work on the Phase Method that I have done in the past, analyzing the Dow Jones Industrial Average index. The supporting Excel file will be called "Phase File 1 Dow phase analysis." We'll graph the Dow index itself on a ***monthly*** basis from 1946 to 2004. The Dow is in column D.



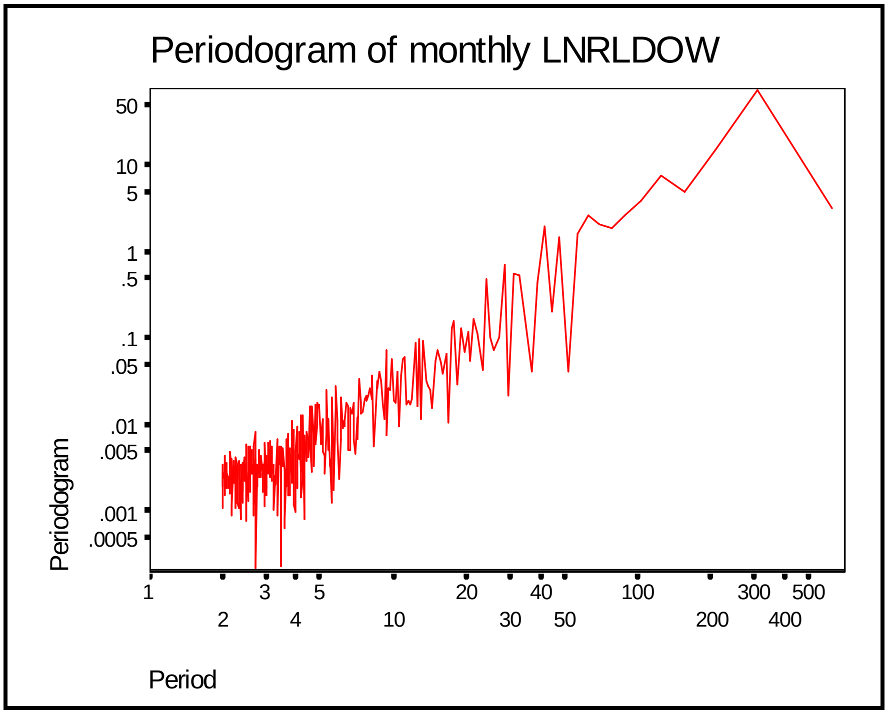
A great deal of the increase in this index simply reflects inflation. In the next graph, based on column G in the Excel file, the Dow index in constant dollars is displayed. The homeostatic motion around a growth trend begins to appear in the graph below.



The graph below is based on Column I of the Excel spreadsheet, and displays the natural logarithm of the Dow index in constant dollars. After taking logarithms, a doubling from 1000 to 2000 looks the same as a doubling from 2000 to 4000. In this graph the homeostatic character of the market is even more apparent; there is irregular homeostatic-cyclic motion around a long-term trend upwards.

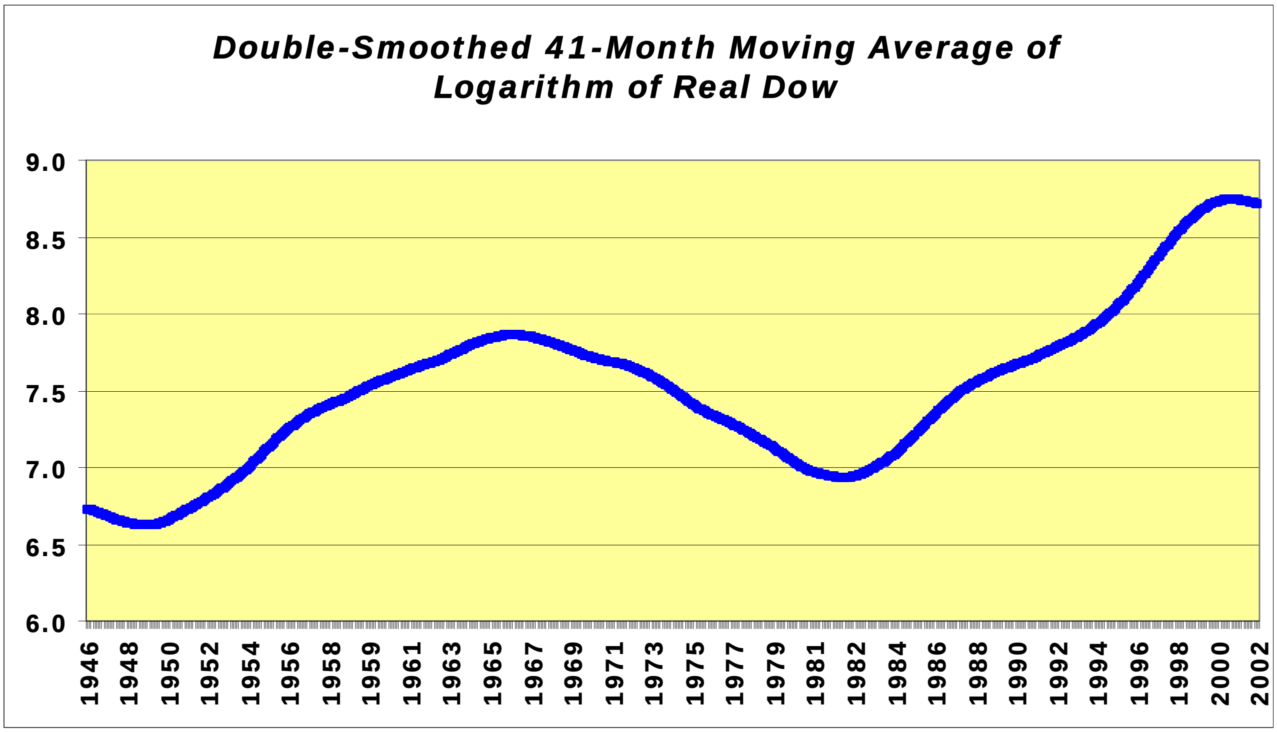


Next, a spectral analysis on this monthly series results in the following periodogram:



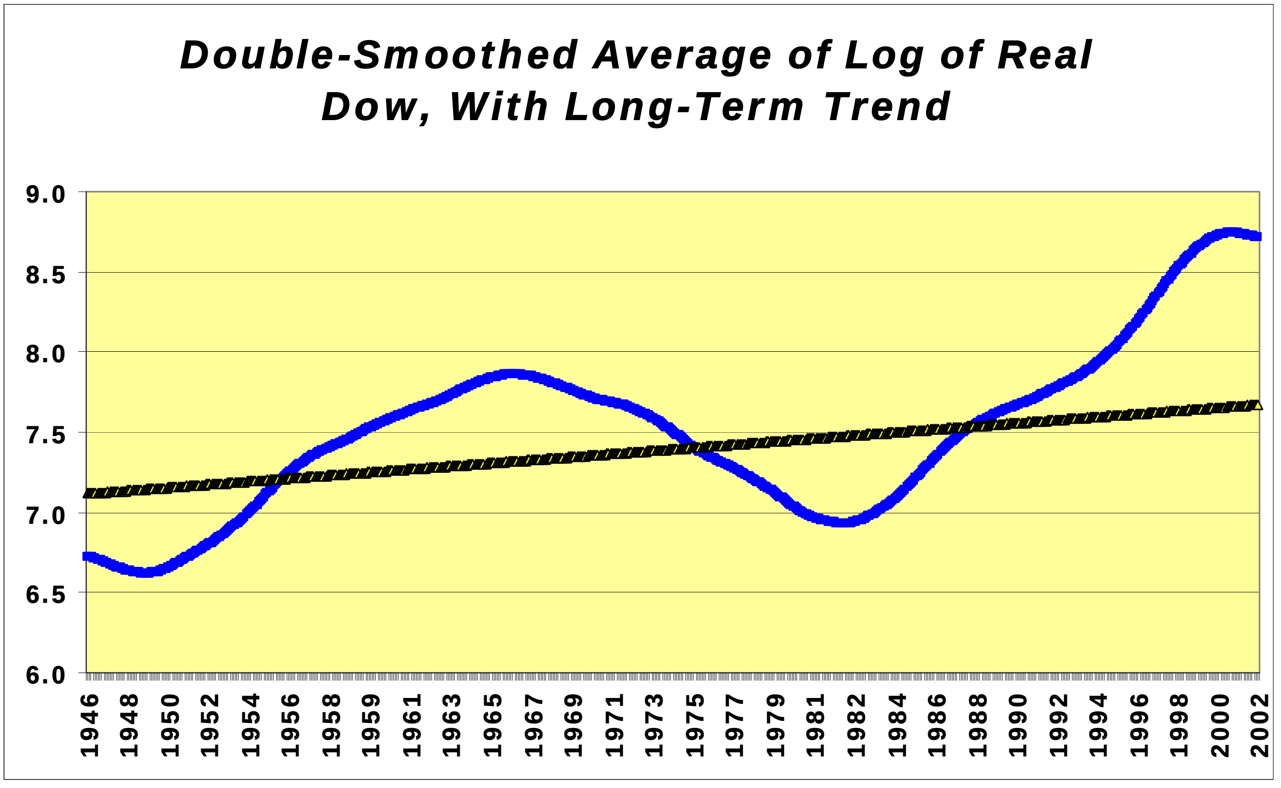
Here we will bypass the peak at 300 to 400 months which represents the ***long*** homeostatic cycle and instead look at the next peak resonance of interest, at about ***41 months*** in length. It will not be necessary to get an exactly correct length, since these cycles are irregular in size, length, and shape in any case – and the Phase Method will produce reasonable results even if the cycle's length is misestimated by a plausible amount.

The aim of the Phase Method is to ***capture*** the long cycle, not smooth it out as integral measure distributions do. Seeing the advantages of the double-smoothed triangular method, we will take the double-smoothed 41-month moving average of the logarithm of the constant-dollar Dow. [The single moving average is in column J and the double moving average in column M.] Note that this cannot be known in real time, but only with a time lag; however, the regularity of its motions make plausible extrapolation possible.

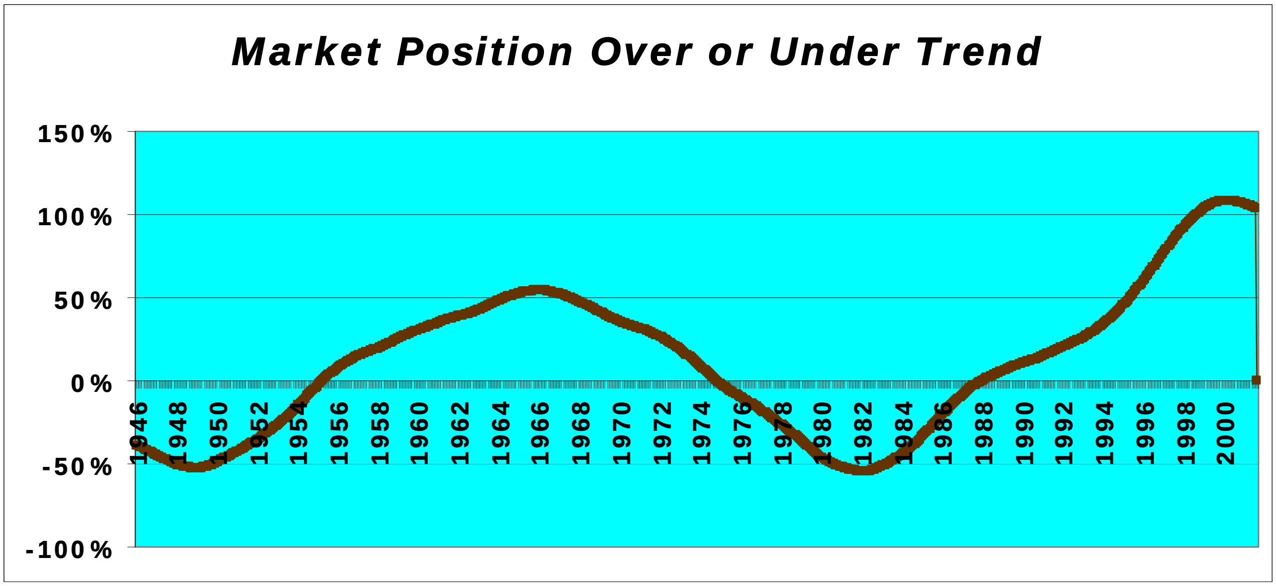


The graph below puts in a long-term trend line, without dividends and without adding inflation back in (column N of the Excel file). How do we build a trend line? One way is to perform a statistical regression and estimate a trend line. Another way is more approximate – to draw a line connecting the "middle points" of the ***previous*** upward and downward moves (which ***are*** known at the ***present*** time). Alternatively, an analyst can experiment with other trend lines, or even use experimental lines to see how things turn out (and whether a proposed trend looks reasonable or not).

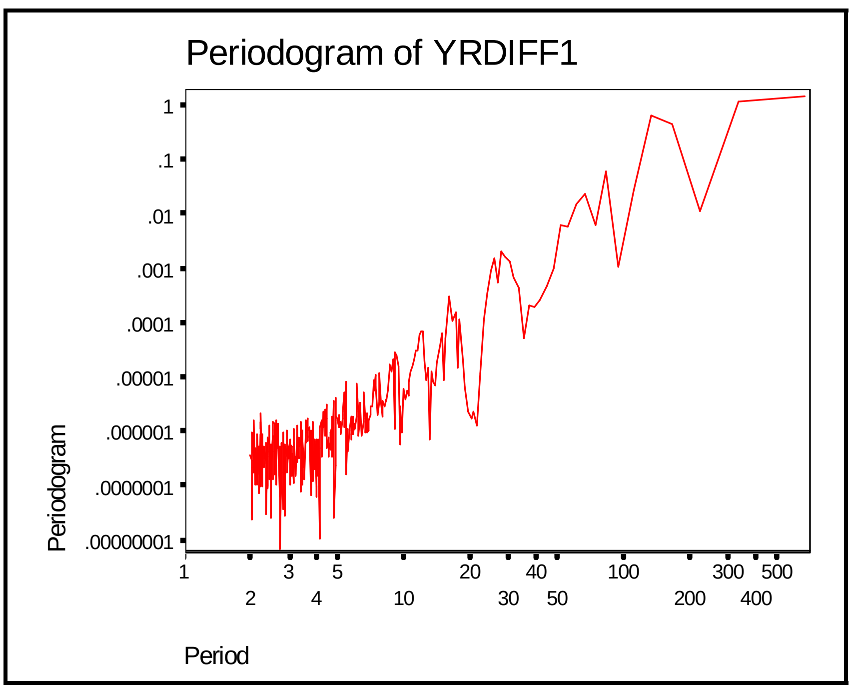
The trend line drawn below may be too flat (the slope may be too small). But it is not necessary to get the trend exactly right; this cannot be done on a current-time basis anyway, but only well after the fact. As we shall see, the Phase Method does not require exactitude to provide a reasonable estimate of the market situation.



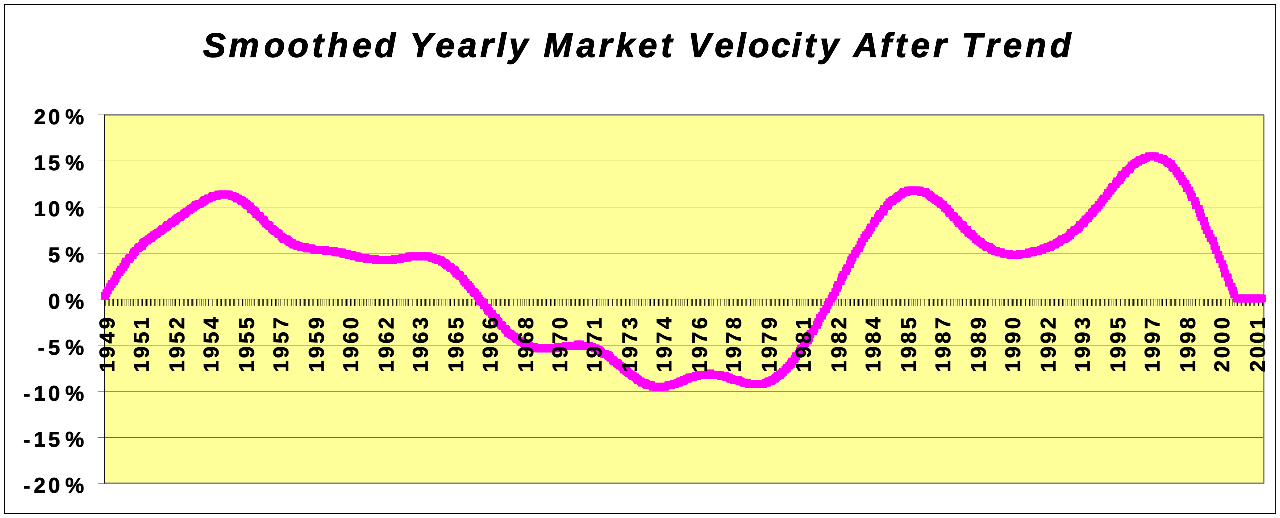
The graph below represents Column P of the spreadsheet. It represents the amount that the double-smoothed 41-month moving average of the logarithm of the constant-dollar Dow is above or below its trend line. This difference will be called the ***market position*** above or below trend. It will be the ***horizontal*** axis of the phase diagram. Perhaps the trend line drawn was not inclined enough, in which case the curve below will be slightly "off." But the market was clearly over its trend line in 2000! This method is ***robust.*** That is, if the trend line is changed (or was slightly incorrect in the first place) the broad character of the graphs drawn and the implications of those graphs are on the whole the same.



The next step is to plot a smooth curve of the yearly ***velocity*** or ***rate of change*** of the above data series. We take the monthly differences and multiply them by 12 – but the resulting series is still irregular (column Q). To smooth it, we'll again do a spectral analysis which produces the periodogram below.



We look for the first resonance that is shorter than the previous 41-month span; this was found at about 29 months or so. Motivated by previous work, we take a double-smoothed 29-month moving average of that series of "yearly" differences (single moving average in column R, double average in column S).



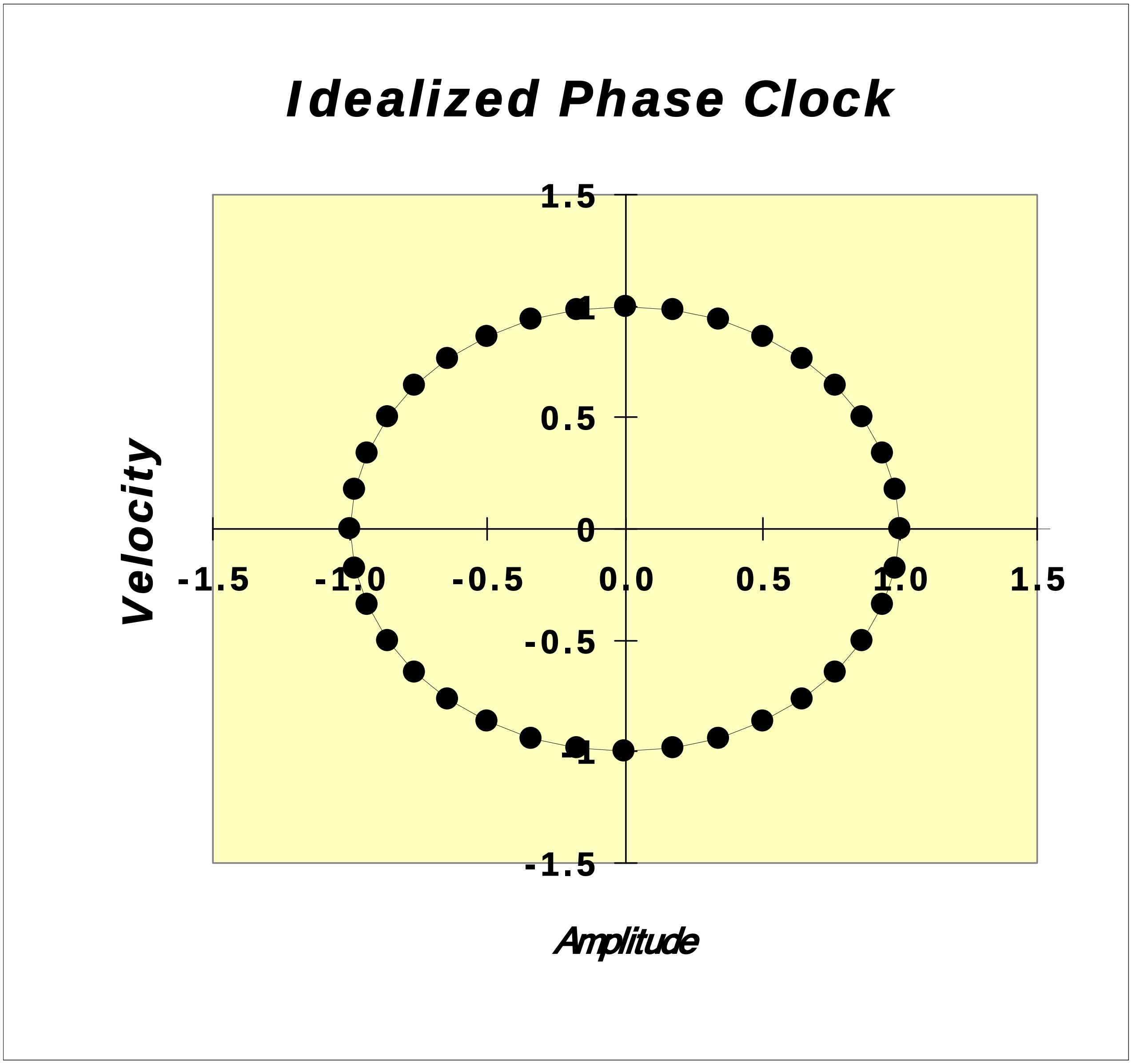
The curve shown above appears smooth enough to use as an estimate of the rate of change (velocity) of the position of the market relative to its trend. This data series will be used as the y-axis or vertical axis of the Phase Clock.

If the business cycle followed an exact sinusoidal pattern, the Phase Clock of a perfect cycle would appear as a perfect circle, going clockwise. In a real-world homeostatic cyclic situation the curve will track out an irregular "round" shape.

The ***horizontal*** axis shows the amount that the (smoothed) market index is above or below its trend.

The ***vertical*** axis represents the rate of change of that horizontal axis data series, smoothed or double-smoothed as necessary.

The graph on the next page shows what the Phase Clock would look like in a "perfect cycle, perfect circle" situation.



At the top of the circle (12:00 on an imaginary clock face) the moving point is above the center but not to the right or left of center. That is, the (smoothed) market is near to its trend (neither right nor left of center) and moving up faster than trend (rate of change relative to trend is positive; moving point is above center).

At the right of the circle (3:00 on an imaginary clock face) the moving point is to the right of center but neither above or below center. The (smoothed) market is at (or near) its maximum above trend, and at that moment is neither rising nor falling relative to trend.

At the bottom of the circle (6:00 on an imaginary clock face) the moving point is below the center but not to the right or left of center. That is, the (smoothed) market is near to its trend line (neither right nor left of center) and moving down relative to trend (rate of change relative to trend is negative; moving point is below center).

At the left of the circle (9:00 on an imaginary clock face) the moving point is to the left of center but neither above or below center. The (smoothed) market is at or near its minimum (farthest) below trend, and at that moment is neither rising nor falling relative to trend.

The Phase Method is adaptable – it is robust – and can face up to a wide variety of real world situations. If the motion of the homeostatic market cycle is large in size (taller waves up and down), the circle (regular or irregular) is larger, but the essential character of the clock motion remains. If the size of the cycles are smaller (the motion is closer to linear trend, with only small waves around it) the circle is smaller, but the essential character of the clock motion remains.

If a cycle (or cycles) is slow (longer in period or length, taking more time to complete), the point moves around the circle (regular or irregular) more slowly, but the essential character of the motion remains. On the other hand, if a cycle is fast (shorter in period or length, taking less time to complete), the point moves around the circle more quickly, but the essential character of the motion remains.

If the motion is imprecise and irregular – as happens in real world situations – the circle is imperfect, perhaps very imperfect, but the essential character of the motion remains.

If the background trend was imprecisely estimated, the circle is off center, but the essential character of the motion remains.

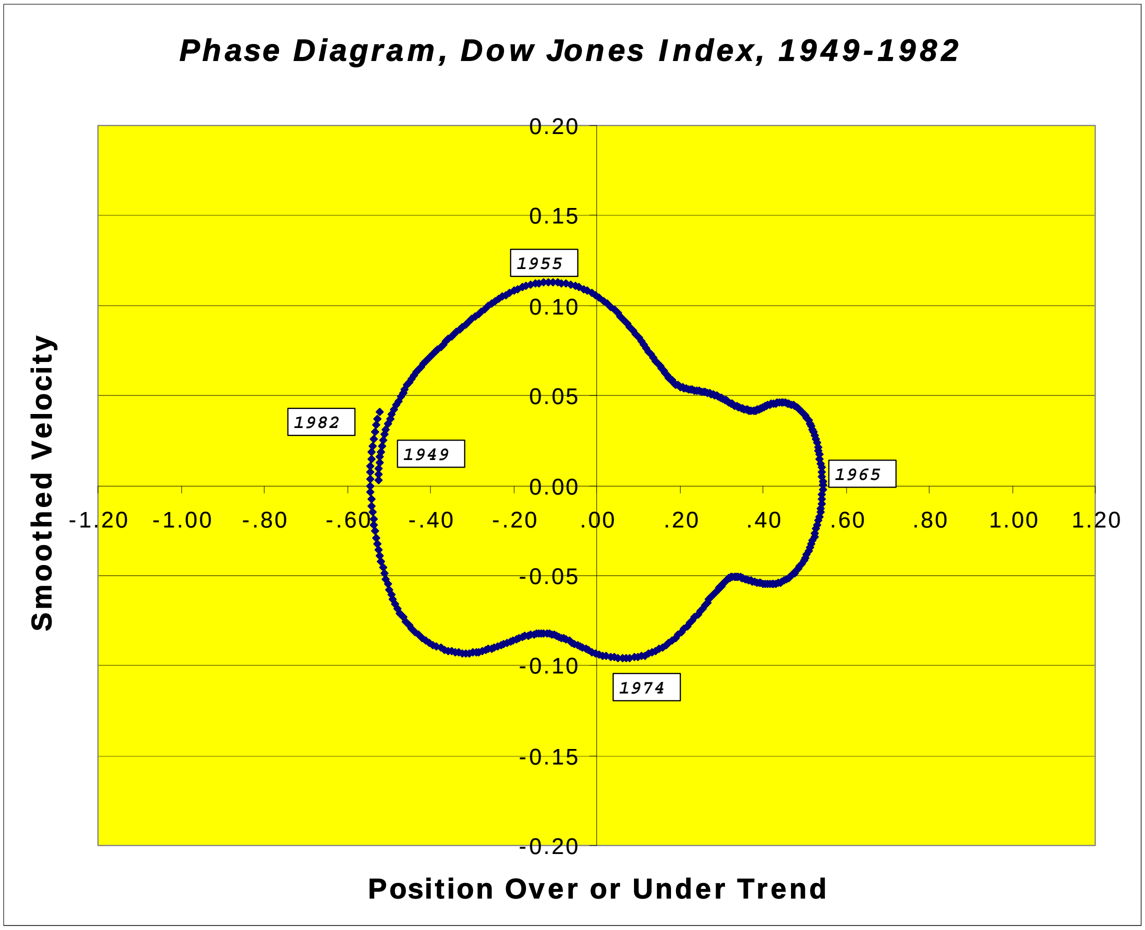
The exact position of the point on the Phase Clock cannot be known in current time because of the time required to complete the computation of the smoothed averages. However, the approximate circular nature and regularity of the motion does suggest a current position, as an extrapolation from known past positions (even based on what is known only up to a few years prior). Thus, the methodology is able to reasonably handle situations of time-lagged information.

The Phase Method can also be used to test hypotheses and theories. That is, the analyst can postulate that the market is currently a certain amount above or below trend, and moving up or down relative to that trend at a certain rate (or is flat relative to trend). In other words, the analyst can postulate a current position on the Phase Clock, and then see if that appears to make sense. The analyst can estimate what would have to have happened, and be happening, for that postulated position to in fact be correct – and can suggest what would be likely to happen in the future if that postulated position was in fact correct. This process can then suggest that the postulated position may be correct, or is very likely incorrect.

The current position on the clock can never be known exactly in current real time. Therefore, I say that the best time to sell on an imaginary clock face is at 1:30 or 2:00, when the market is definitely above trend and still moving up – rather than trying to catch the perfect exact top of the market at 3:00 on the clock face. People who wait for the precise top often wind up waiting too long!

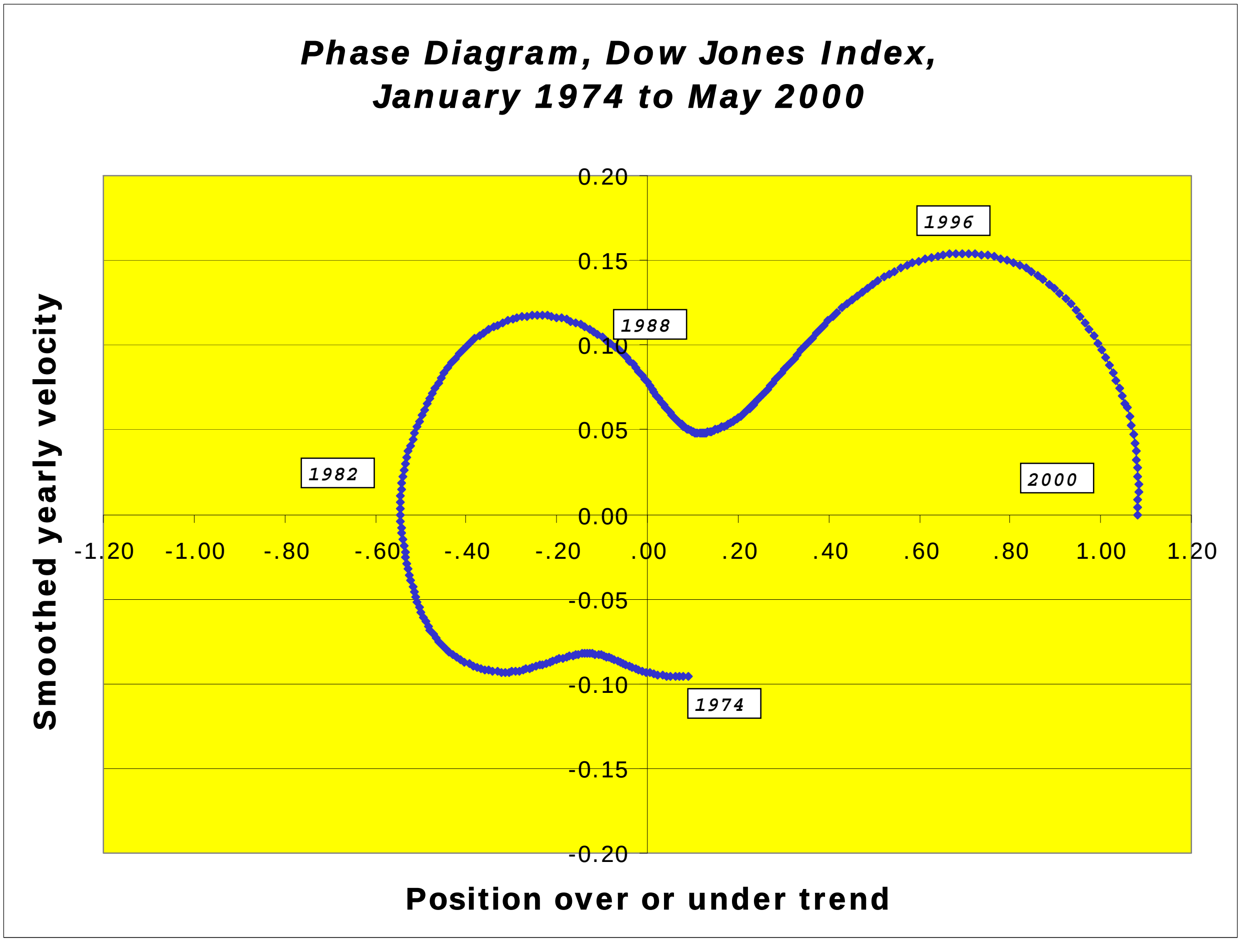
In the same way, a wise time to buy may be when the phase clock is at 7:30 or 8:00 on an imaginary clock face, when the market is definitely below trend but not too far from its bottom – rather than trying to catch the perfect exact bottom of the market at 9:00 on the clock face. An alternate approach would be to buy at 10:30 or so, when a rising market seems to be established but there is still a great deal of gain to be captured.

The graph on the next page shows a Phase Clock for the Dow Index as smoothed earlier in this paper. The x-axis (horizontal) represents how far the smoothed curve is above or below trend, and the y-axis (vertical) shows how fast this smoothed curve is moving faster than trend or falling behind trend. If the trend line was not quite right, the clock will be "off" but the general "look" and "message" of the graph will not be far different. As has been stated, the position of the "moving point" cannot be known exactly in real time, but the rough and approximate circular shape of the graph suggests that extrapolation around the curve is worthwhile. In "Phase File 1 Dow phase analysis" the horizontal axis is in column U and the vertical axis in column V.



Inspecting the graph above, allowing for the time-lagged nature of the information, we can still see that the Phase Method functioned well in the different parts of the homeostatic cycle. Knowing where the market was in the past (even a few years in the past), we can use the fairly regular character of the motion combined with reasonable extrapolation (perhaps using shorter moving averages and some estimation) to extend the graph and suggest current-time position or near-to-current-time position, at least on a trial basis.

The graph below continues the Phase Clock diagram into a more recent time period. There is still some extrapolation in that graph, but it is difficult to argue with the basic appearance of the curve – and its judgment that the market was well "above trend" in 2000.



The Phase Method looks not at a market index (such as the Dow) itself, but rather at a transformed version of that index – strictly speaking, at two indices (horizontal and vertical axes) which themselves are smoothed, or double-smoothed, moving averages of the Dow or other market index. Thus, the Phase Method and its Phase Clock will smooth over a great deal of motion up and down over terms of days, weeks, months, and even a year or two. These motions can be of considerable size – commonly 10%, or 1000 points on the Dow, and sometimes quite a bit larger. Many in the investing community and the public at large find themselves following these motions rather than any long-term trend or long-term cycle. These motions get a great deal of attention and cause emotional excitement and depression. They can constitute tradable rallies or dips. The Phase Method misses them all and looks at a very large-scale overall picture. The Phase Method provides no help for short-term trading and little help for most medium-term investing. However, it is very useful in understanding long-term, broad homeostatic cyclic motions.

An analyst in the middle to late 1970s and early 1980s using the Phase Clock (even allowing for time lag and making some extrapolation) would have considered that those years were probably a good period in which to invest – and these years were times when many shied away from the market – though in retrospect they turn out to in fact have been good years to invest! The Phase Method's insights turned out to be right in the long run, after a period of several years of embarrassment and apparent refutation and frustration. [This is a very common result of the use of the Phase Method.]

The Phase Clock applied in 1987 (with the curve known to a few years earlier, and reasonable estimates of current market position) would have suggested that the market was not as grossly overextended as it had been in 1929 (or was to be in 2000), and thus the crash of October 19, 1987 was to be considered as a short-term or middle-term dip or correction rather than the beginning of a decade-long bear market phase.

The Phase Method would have warned, and did warn, that the market was becoming overextended in the late 1990s. In my 1998 published paper I did just that. It appeared at the time to be a foolish warning. The market continued to rise. Traders and speculators made a great deal of money for a time. However, in the long run, the general advice of the paper turned out to be reasonable. Logarithmically speaking, the great majority of the bull market gains had already been made (in percentage terms), although the market later reached new highs (but several years later).

Now let's look at some quotations from my earlier papers, describing some of the key points of the Phase Method. First, the Phase Method emphasizes robustness. It can function reasonably well if the data is time-lagged or if the analyst makes misjudgments. It can still work right even if it is somewhat wrong.

> "A robust model is needed…It must hold up even if the analyst misjudges things a bit" (Cagan, 1991 paper, p. 208).
>
> "Clock time is a good guide to the investor. He won't be able to catch the last dollar but will avoid being grossly burned" (p. 209).
>
> "A Dow of 4000 to 6000 (before allowing for inflation) would not be impossible by 1994 to 1996" (p. 214). Written in 1990, published in 1991.

Looking back, I don't regret having made the above statements.

As has been suggested, the Phase Method many times offers contrarian suggestions that seem and feel unpalatable at the time – and are frequently wrong for months or years.

> "The phase method often gives contrarian advice, such as sell signals at the top of a market when everyone is optimistic…
>
> The phase analyst must be willing to be called a fool, and even to actually ***be*** a fool, for quite some time – but this is no disgrace if he is proved right in the end!"
> (1998 paper, p. 211).
>
> ***"What about all the profits being made in the current bull market?*** In most cases, they are simply being made on paper – or in a file on a computer disk…The only way to take advantage of these profits is to take money out of the market and [invest it] elsewhere…Otherwise, ***many could experience a rise from rags to riches on paper only, not receiving their hoped-for profits when they sell"***
> (1998 paper, p. 211.)

Looking back, I don't regret having made the above statements.

As been stated, the Phase Method looks at the long-term motions of the homeostatic cycle and misses a lot of motions in the short run and even in the middle term. It suggests a long-term bear market will come in the next decade or two. I suggested this was in harmony with demographics:

> "Much of the rise over the last few years has been due to large inflows of money invested in the market, often for retirement purposes…What will happen in the period from 2010 to 2030, when the large Baby Boom generation is expected to withdraw its money from the market for their retirement needs?...This could create a downdraft on the market as powerful as the updraft that was created when they put their money in….Further, younger generations and foreign investors might be reluctant to take up the slack by investing trillions of dollars in the market in the face of this downward influence….Many Baby Boomers would wind up receiving much less for retirement than they expected. The looming entitlement crisis associated with 2010-30 may well lead to large government deficits and a rekindling of inflation. In such a case, everyone could receive their retirement money in nominal terms, but with its actual buying power severely reduced"
> (1998 paper, p. 211).

Looking back, I don't regret having made the above statements.

The Phase Method also has an interesting way of visualizing (visually displaying) real or alleged ***bubbles*** in a market. I define a ***bubble*** as distinct from the normal business cycle with its positive and negative phases. A market can go through a complete business cycle with prices going over and under trend, without a bubble having occurred. A market is in a ***bubble*** when it functions not according to economics, but is rather driven by its own momentum and psychology, often of an extremely optimistic type.

A classic sign of a bubble, if it exists, is when prices get farther and farther above their long term value or trend – and yet rise faster and ever faster! ***Any*** economic model, including models of the business cycle, would suggest that prices that are out of long-term equilibrium will experience a restoring force (although that force may take time to have its full effect). The further away from "center" a market gets, the stronger should be the pull backwards, and prices should sooner or later slow down in their ascent, level off, and finally begin to decline, thus starting a new bear phase that may well overshoot the long-term trend on its way down. No matter what econometric equations are used to model this motion, there is no purely economic model that teaches that prices should rise faster and faster the farther and farther away they get from their "normal" valuations.

Such runaway momentum in a market does not arise from the laws of economics but rather from the realm of psychology. People get excited about the gains that they or others have made in a bull market. The positive psychology takes on a life of its own. Greater and greater gains are made as people take greater risks and pay higher prices. Most if not all doubters, except for a few contrarian cranks such as me, are refuted and crushed as the market breaks one ceiling after another. The "efficient market" concept does not arise in such cases, because the market is not functioning as a classical "market" so much as a single "group mind." Market prices can rise to unbelievable levels that have nothing to do with real economic valuations, because the market is now driven by psychology instead of economics. Examples include the tulip bubble in Holland in 1636-1637, the stock market bubble of 1929, and the Internet stock bubble of 1999-2000. In 1999 to 2000, the market "valued" companies at billions of dollars that had no earnings and sometimes almost no revenues. In the tulip bubble, people bought tulips (and tulip futures) with no intent of holding them, and sometimes without actually taking possession of them at all, but only to speculate (the Dutch word is ***windhandel,*** literally "trading in the wind"). In the tech stock bubble, people bought stocks and sold them days or minutes later without knowing or caring what the companies actually did. Those markets were bubbles, driven by momentum and psychology.

Graphically, a bubble is marked by a point of ***inflection*** in pricing, in which a price graph that would be expected to slow down instead reverses direction and "takes off" ever higher and ever faster.

On the Phase Clock, a bubble is manifested as a bulge – literally, a bubble! – in the rightmost part of the circle. The distortion is visible. By this test, the stock market in the 1960s went through a bull market phase and peak, with little "bubble distortion." In contrast, the Dow experienced greater distortion in the late 1990s. Please see the phase diagrams exhibited earlier in this paper. Had a phase plot been constructed for the NASDAQ index the distortion would have been even larger.

The reader cannot help but notice that the Phase Method has been skeptical regarding the stock market for the last ten years or so. I still believe that the stock market will resume a bear market phase, particularly after inflation – looking over the next fifteen to twenty years or more. ***Economic*** reasons include the likely fall in the dollar relative to stronger world currencies, due to our large trade deficit and budget deficit and our monetary expansion; and the coming retirement of 68 million baby boomers, with the consequent strains on our society and the likelihood that they will wish to or need to withdraw much of the money they had invested in retirement plans. The Phase Method ***itself*** merely suggests the graph will round the curve and move down. The Phase Method, of course, shouldn't be used in a pure vacuum. It should be used together with existing economic techniques, such as analyses of price/earnings or price/dividend ratios, etc. But it sure is nice!

Of course, the ***very*** long term trend ***is and remains upward*** (barring an asteroid impact, a nuclear war, an ecological collapse, or some other horrible event that destroys our country and moves the long-term trend from positive to flat or negative).

It is the integral investment measure plan suggested earlier in this paper that provides ***a way out!*** We have seen that these measures (plans of buying and selling) provided a good long-term return, ***no matter when they were started,*** because they are broad enough and long enough to straddle the bull and bear market phases and capture the long-term return. These push-pull plans have been shown (historically) to yield a good long-term minimum return whether they were started in the bullish times of the 1920s and 1960s, or whether they were initiated in the depression of the 1930s.

In this way we see there is no inconsistency between the current bearishness of the Phase Method and the long-term optimism of integral investment measure plans. We can soberly anticipate a coming bear market, ***and yet still*** offer the younger generation a historically validated plan that can reasonably be expected to yield them a decent and robust minimum return. These ideas could provide the basis for add-ons to Social Security or a private retirement plan for younger workers (it's too late for those already near retirement), and permit the offering and sale of a great variety of robust long-term investment plans. These breakthroughs could clearly be helpful to our society.

NEW IMPROVEMENTS IN PHASE METHODOLOGY: A TECHNICAL STUDY

Next, I propose to use the Phase Method to expand the mathematical sonar from "functional, parametric, genetic" to a ***four-step*** methodology:

Functional
Parametric
Genetic
Informed.

In this part of the development, the Phase Method and its Phase Clock, however inexactly known (regardless of whatever time lag is involved in knowing the correct position) is used to ***inform*** a previously constructed integral investment measure plan and suggest modifications to it.

Because of the approximate and inexact information provided by the Phase Method, only a very coarse and conservative "informative" methodology will be suggested. But even modest "Phase Method informative" adjustments will produce very useful gains in minimum return and median return.

All investors would like to buy low and sell high. The Phase Method provides an approximate navigational guide to how the market homeostatic cycle is progressing. To allow for the inexact nature of the method, we will define the market as "high" when it is ***40% or more over trend,*** and also require that a bull market have been established for ***four years or more.*** The market will be defined as "low" when it is 40% or more ***below*** trend, and when a bear market has been established for ***four years or more.*** This coarseness and conservatism is used in an attempt to allow for any inexact estimation due to an incorrect trend line, time-lagged phase information, and so on. These definitions are clearly defensible and very conservative; in fact, they may be criticized as being too conservative.

According to this method, the market would be classified as "high" in 1928 and 1929 and as "low" in 1934 and 1935. It is difficult to argue with such a judgment, as many contemporary observers said the same thing even as they rode the waves of excitement and depression. In fact, it could well be said that this coarseness is far too conservative, since the "four year establishment" requirement ruled out buying in the depths of the depression during 1932 and 1933!

According to this method, the market would be classified as "high" from 1965 to 1969 and as "low" from 1979 to 1984. Again, it would be difficult to disagree with such a judgment. Almost all observers of those times felt the same way – but many found it psychologically difficult to sell in a bull market or buy in a bear market. It was psychologically easier to ride along with a bull market (waiting for an absolute top, which is in fact impossible to spot), or to stay out of a bear market (waiting for it to turn, often waiting too long). If you disagree with my choice of "high" and "low" years, by all means make modifications and recalculate – but you will still see the basic advantages of informed phase planning.

According to this method, the market would be classified as "high" from 1998 to 2000. Again, it would be difficult to disagree with such a judgment, since almost everyone felt the same way at the time – even as they rode the market to ever more dizzying heights. In fact, again the accusation of excessive conservatism might be made – that the succeeding years should also be called "high." However, since the market had fallen since its 2000 peak, we will reject that temptation. As we shall see, even a small amount of "informative" adjustment – that allows for the imprecision of phase analysis, and is difficult to argue with – produces quite a bit of gain. This ***informative*** use of the Phase Method will be applied to the ***mechanical plans of integral investment measures*** to produce hybrid arrangements that yield a better minimum return than the mechanical plans, using only a minimal, conservative and defensible form of market judgment.

We will begin with the "truncated 40 years wide lag 25 years" plan that has already been developed. But this time we will introduce the following ***informative*** corrections, informally called a "double or nothing" plan.

In years where the market is "high" do not buy the Dow index at all.

In years where the market is "low" buy twice as much as the prior plan had called for – whatever that was.

In years where the market is neither clearly "high" or "low," do what the prior plan specified.

Let's make ***no adjustment at all*** to the selling process. In other words, each year's purchase will be "robotically" sold 25 years later, ***with no attention to whether the market is at that time "high" or "low."*** Also, each year's purchase will be completely sold in its exact quantity, whether that quantity was the same as in the prior plan, or whether it had been doubled, or whether it had been reduced to zero.

This is a very elementary – almost stupid – ***informative*** adjustment. It results in the total weights sometimes adding to more or less than 1, but it is still possible to compute the annualized return. Please see the Excel spreadsheet "Phase File 2 Truncated 40 Yrs Wide Lag 25 Yrs Phase Informed Buying Only."

The plan of the spreadsheet will be different than in previous files as we shall now see.

In this file, the real constant dollar Dow is in column F as before.

The raw weights are in column O and they are normalized in column Q so that they add up to 1.

The rows representing "high" years are colored yellow.

The rows representing "low" years are colored blue.

The ***revised*** weights start in column R and go up to column BT. These are the weights from column Q, but reset to zero in high (yellow) years and doubled in low (blue) years – from whatever their previous plan called for them to be!

The starting years of the distributions will go from 1910 to 1964.

The ending years of the distributions will go from 1950 to 2004.

Columns BW to CZ represent the real Dow multiplied by the ***revised*** weights, as ***informed*** by the Phase Method. The real Dow is from column F (this column does not change). The revised weights start in column R and move one column at a time to the right. The resulting products will represent the expenditure in constant dollars for the ***buying*** or ***investment*** phase. Since this is the buying phase, we need only consider starting years from 1910 to 1939 and ending years from 1950 to 1979. The selling distribution will be set 25 years forward from these years.

Columns DC to EF represent the revised (informed) weights, multiplied by the ***real Dow 25 years ahead.*** The revised weights come from column R and the columns to its right (one column over at a time) and the real Dow 25 years ahead comes from column L (this column does not change). The resulting products will represent the money received in constant dollars for the ***selling*** phase.

Because of the changes made by the ***informative*** methodology, the buying distributions are ***not*** the same for different starting years. This is why we have to separate the columns for the value of the buying distribution from the columns for the value of the selling distribution, for each starting year, even though the selling distribution is the same as the buying distribution (only shifted 25 years).

Cells BW111 to CZ111 represent the total sum amount in constant dollars associated with the ***buying*** distribution for the different starting years. Sometimes the total weights added up to more than 1 or less than 1 (because of the "informed" changes), but we will correct for this.

Cells DC111 to EF111 represent the total sum amount in constant dollars associated with the corresponding ***selling*** distributions.

Row 113 represents the 25 year return. It is constructed differently than in previous files. For instance, cell BW113 represents DC111 divided by BW111. Note that this automatically corrects for the total weights possibly adding to more or less than 1 due to phase-informed weighting. The same thing happened in the selling distribution, column by column as happened in the buying distribution; and so it cancels out. The 25 year returns are in cells BW113 to CZ113.

Dividing the appropriate natural logarithm by 25 yields the annualized returns in cells BW115 to CZ115.

The minimum, median, mean, and standard deviation of the annualized returns are in cells BW117 to BW123. This is copied into P39 to P45. The minimum and median annualized returns are also copied into cells C1 and E1.

The minimum annualized return has risen to 1.539% and the median annualized return to 2.203%. This represents a ***considerable*** improvement over the prior plan in "File G2," with a very simple level of "informed" changes to the previous plan.

A more advanced ***informative*** adjustment would be to modify the ***selling*** measure distribution as well as the ***buying*** distribution. It will then be necessary to consider the cumulative amounts purchased so that one does not sell more units of the Dow than he or she has previously bought (omitting the possibility of "short selling"). Since the selling plan lags the buying plan by 25 years, such an eventuality will be infrequent, but we must allow for its possibility. Consider the following very simple set of rules, added to the previous rules:

If the market is "low" do not sell any previously purchased units of the Dow.
If the market is "high" sell twice as much as the prior plan had called for.
In years not clearly defined as "high" or low" do what the prior plan had called for.
If at the ending year (as the prior non-informed plan would have called for) you still possess some leftover units of the Dow (due to the balance or imbalance of "high" and "low" years), sell everything robotically at the last year and close the plan.
If at any time during the plan you run out of Dow units (due to previous years of zero purchase when the market was high, and current years of double sales when the market is again high, for instance), sell what you still have and close the plan and never reopen it – we'll call this a "foreclosure."

Again, this informative plan is still elementary, almost stupid. Its results appear in the Excel spreadsheet "Phase File 3 Truncated 40 Yrs Wide Lag 25 Yrs Phase Informed Buying and Selling." In this analysis it is all the more crucial to separate the selling distribution and its value from the buying distribution and its value, since the two distributions themselves may have very different weights.

In this file the real Dow is still in column F and its 25-year future is still in column L.

The unrevised ***buying*** weights are in column N.

The ***revised buying weights*** (either left as they were, or set to zero, or doubled) are in columns O through BQ. I have studied the starting years from 1910 to 1964 and the ending years from 1950 to 2004, although some of these will be not necessary to consider as ***buying*** plans for the purposes of this analysis. However, we ***will*** need these when considering the ***selling*** distributions!

Columns BT through CW represent the constant dollar value of the buying distribution. We multiply the real Dow in column F (this column does not change) by the revised buying weights in columns O through AR (shifted over one column each time). Since these columns only consider the buying distributions, we stop with column AR for that multiplier, representing a buying distribution starting in 1939 and ending in 1979. As in the previous Phase File spreadsheet, we will look at ***buying*** distributions with starting years from 1910 to 1939 and ending years from 1950 to 1979.

Columns CY to EB represent ***revised selling weights.***

These have been peremptorily set to zero during the "low" years of 1934 to 1935 and 1979 to 1984.

These weights have been doubled during the "high" years 1928-1929 and 1965-1969 and 1998-2000. The doubling is done ***from the "revised weights"*** of columns O through AR.

The weights have been left unchanged during years that are neither "low" nor "high."

Columns FI to GL represent the ***cumulative buying weights*** – how many Dow units have been cumulatively bought to the current year.

Columns GN to HQ represent the ***cumulative selling weights*** – how many Dow units the plan calls for to have been cumulatively sold to the current year. It will be important to compare the cumulative buying with the cumulative selling.

Columns HS to IV represent the ***cumulative holdings*** – the number of units bought minus the number of units sold. If the possibility of "short selling" is excluded these holdings must never be negative. Since the buying distribution is 25 years ahead of the selling distribution, imbalances will be rare, but due to the intervention of high and low years, imbalances are possible.

If imbalances occur – if the cumulative holdings are prescribed to go negative – the plan is to be "foreclosed." That is, the remaining holdings must be sold out during the last year that net holdings are positive, and the plan not reopened. The net holdings will then remain zero for all subsequent years.

In fact, I ***already made*** the required adjustments in columns CY to EB, adjusting the last yearly cell in each selling distribution that had to be "foreclosed." This is noted in columns CY to EB. This is also the reason there are no negative numbers in columns HS to IV – because the necessary foreclosure corrections ***have already been made.*** Why didn't I have more columns – to show first the imbalances, then the necessary corrections, and ***then*** the corrected selling weights? The problem is that Excel only allows 256 columns and ***there was no more room!*** Therefore, I had to declare in text what needed to be done, do the spreadsheet with whatever imbalances (negatives) showed in columns HS to IV, and then adjust the final selling weights in the last years of the selling columns CY to EB to represent foreclosure sellouts that brought all imbalances to zero – as they now appear! The adjustment work has thus receded into the "background," as work "already done," but the specifications clearly describe what had to be done – and what therefore was actually done. A longer analysis could use more space to display more detail.

Let us return to columns CY to EB. Some of the selling plans are "foreclosed" before their planned time; see cells CY74, CZ74, DA75, and so on. When an apparent imbalance (negative) was called for (to sell more than had been bought), the selling weight just ***above*** the word "foreclosure" was adjusted to sell out (then) the exact quantity still held, which foreclosed the plan (not to be reopened).

Some of the plans did not have any such ***negative*** imbalances. In these cases the revised selling weights (whether zero, doubled, or left intact from previous columns) were retained. When the plan's selling years came to their end as had been planned, all holdings were peremptorily sold out and the plan marked "closed." See cells DQ98, DW104, etc. Note that in some columns the selling weight immediately ***above*** the "closed" text is larger than one would expect, indicating the peremptory selling-out of remaining holdings when the plan is closed according to schedule. In this way ***positive*** imbalances of holdings over sales were resolved when the plan was closed in its prearranged year.

Note that all imbalances, negative and positive, have been (already) rectified through the application of the rules.

Note that the possible problem that the total weight of purchases is not equal to the total weight of sales has been solved. By the rules of the plan, all purchases get sold by the time the plan ends. The amount purchased and the amount sold are equal.

Columns ED through FG represent the value of the ***selling plan*** in constant dollars, after any and all adjustments (including doublings, zeroings, closures, or foreclosures) have been made. The revised selling weights in columns CY to EB are multiplied by the (fixed) column F, which represented the real Dow.

Cells ED111 through FG111 constitute the sum of the numbers above them – the total value of the selling distribution in constant dollars.

> Cells BT111 through CW111 constitute the sum of the numbers above them – the total value of the buying distribution in constant dollars.
>
> Now we can compute the 25 year return. Cell BT113 is cell ED111 divided by cell BT111, and so on to the right, all the way to cell CW113.
>
> Then row 115 (cells BT115 to CW115) represents the annualized return, taken by dividing the appropriate natural logarithm by 25.
>
> The minimum, median, mean, and standard deviation of the annualized returns in constant dollars appear in cells BT117, BT119, BT121, and BT123. They have been copied for the viewer's convenience into cells M39 through M45, and the minimum and median annualized returns also appear in cells C1 and E1.

Even the very conservative informative adjustment suggested above raises the ***minimum*** return all the way to 2.339% and the ***median*** return to 2.865% – a ***major*** improvement!

But even this does not represent the full return of the informative method. In years when a plan was foreclosed, it was assumed that the total output was then held idle (with no interest or appreciation) until the previously planned closure year – and then the annualized return was calculated. In fact, the output will be invested somewhere, perhaps in conservative corporate bonds or Treasury bills. The true return in foreclosed plans will be ***larger*** than the numbers calculated in "Phase File 3."

One way to estimate the true return due to the informed plan itself in foreclosure years is to look at the annualized return as adjusted for the shortened number of years (when foreclosure happens). This was done in "Phase File 4 Truncated 40 Yrs Wide Lag 25 Yrs Phase Informed Buying and Selling Allow Foreclosure Time."

> Here cells CY109 to EB109 represent an attempt to evaluate the number of years "early" that a plan had to be foreclosed prior to its prearranged closure. When the plan was closed normally, the corresponding number is zero.
>
> Subtracting half the "109" row from 25, cells CY118 to EB118 represent a "true length" of the plans whether foreclosed or not. The reason I subtracted only half the foreclosure time was because the cutoff was done only on the closing end. If a plan was foreclosed 2 years early, it is reasonable to try considering its true length as 24 years. Cutting the true length to 23 years is probably unreasonably optimistic.
>
> Then we revise the annualized return in cells BT115 to CW115 to allow for a possible shorter time of the plan. Instead of dividing the natural logarithms by 25 we divide them by the corresponding number in cells CY118 to EB118 whether than number is 25 or less than 25.
>
> The annualized return remains the same for plans closed normally. For plans closed early the annualized return increases since the time length of the plan is less than 25 years.

In this file the minimum annualized return remains at 2.339% but the median annualized return has risen to 2.956% – almost three percent without dividends and after inflation (see cells BT117 and BT119). With a historical average dividend rate of 3%, this would be about ***six percent after inflation!*** Allowing two to four percent inflation, this would be ***eight to ten percent per year*** in current dollars. Considering this as a minimum or robust return, it represents a considerable improvement on the current state of the art regarding minimum returns. Moreover, as mentioned earlier, the return may be even higher, since dividend yield tends to be larger when the market is low. If dividends are reinvested this provides an additional investment when the market is low and future appreciation is likely to be high. Conversely, when the market is high, dividend yield tends to be low, thus reducing the contribution due to dividend reinvestment, at a time when future appreciation is likely to be lower than normal.

Now, the truth probably lies somewhere between Phase File 3 and Phase File 4. In any event, informed plans appear able to attain a minimum annualized return of 2.3% or more and a median annualized return of 2.8% or more – without dividends and after inflation. The breakthrough advance has been made!

In the preferred form, the ***informative*** element is the known or estimated, past or present, Phase Clock position. In alternative versions, the informative element may be supplemented by other indicators such as price to earnings ratios, dividend yield (dividend to price ratios), demographic information, or other inputs. Obviously, high earnings and dividends relative to prices can make stocks attractive. Demographic data can also be informative. For example, people between 35 and 55 years old are usually in their peak investing years. Their incomes have risen over the years, so they have money to invest. They are thinking about their retirement, and are open to investing. But they are not retiring immediately – they do not need their money right away – and so they are often willing to tolerate low earnings and dividend yields in the hopes of future price appreciation. Also, because they do not need their money right away, they are often willing to overlook price declines (disappointments) in the expectation that things will turn out well by the time they ***do*** need their money. Demographically, then, the presence of a high proportion of 35- to 55-year-olds would be favorable to market investment and act as a positive influence on prices. In recent years, our country ***has*** indeed enjoyed this demographic situation with the large Baby Boomer generation experiencing their prime earnings and investment years. In future decades, the presence of a high proportion of Baby Boomer seniors interested in drawing upon their investments for retirement needs may exert a negative influence on the market. In any case, demographic indicators may act as additional informative elements, supplementing Phase Clock information, in alternative versions of these plans. Furthermore, in alternative versions, still other inputs may be used from a wide span of economic and other sources of information.

Before proceeding, we should make several observations. First, the ***informed*** plans are better suited to individual investors or to groups of investors, more than to a governmental plan. It would be difficult for an entire nation (or a governmental plan) to avoid buying at all for a few "high" years, and to buy intensively in a few "low" years. In fact, such changes might be enough to affect the market itself – which might not be entirely bad after all.

Second, while informed plans are numerically very attractive, they can be ***psychologically difficult*** to actually execute. The "pull" in favor of buying during "high" years after a bull market is established for four years or more is very strong, especially as the market continues to rise ever higher – and yet this is when one ***should*** be selling. On the other hand, people "know" they should be buying during "low" years but no one wants to do so. To increase one's buying after a bear market is established requires an intense ***contrarian*** effort of the will – especially as one is proved wrong over and over again. The same is required to sell into a seemingly unstoppable bull market – especially as one is proved wrong over and over again. Many contemporary observers saw correctly that the bull markets of 1928-1929 and 1998-2000 could not continue forever – and yet few people actually sold out and withdrew their gains. The hypnotic effect of seemingly endless gains, combined with the "herd mentality" that can permeate a business community and even a popular culture, makes it very difficult to fight against the tide, especially when it seems to be wrong – and all the more when it ***is*** wrong. As I said in a paper, the phase method analyst must be willing to be ***called*** a fool for years – and to actually ***be*** a fool for years at a time, before being vindicated in the end.

Third, it is possible to start with these informed plans and ***return to functional, parametric, and even genetic methodologies.*** For instance, the idea of "double or nothing" could be readily modified. A simple parametric modification would be to modify the "2" in "double" and the "zero" in "nothing." The "truncated triangle" function could be modified as well, whether functionally, parametrically, or even genetically. However, one must be careful not merely to "chase the noise" in the data series without actually making a clear and understandable improvement in strategy that is likely to work in the future as well as in the past.

Fourth, informed plans also present a temptation to ***market timing,*** with the attendant possibilities for incorrect buying and selling. The more "freedom" or "independence" is found in an informed plan, the easier it is for an investor to be "pulled" one way or another, sometimes into a mistake. There is always a temptation to throw caution (and abandon previous plans) to the wind in a bull market, and there is always a temptation to withdraw from the market (and abandon previous plans) in a depressed situation. The plans suggested in the present paper represent an attempt to strike a balance – to capture the gains of informed buying and selling while still retaining much of a mechanical or objective character – so that they can be set up to run by themselves, to resist temptation, and to grind out a good robust minimum return. For this reason, I have proposed only elementary, almost stupid, informative modifications of the preceding truncated triangle plan. These modifications are difficult to deny or argue with, yet they provide real improvements in minimum and median return. Investors will be strongly tempted to introduce greater modifications – and more power to them if they're right! They can reap even greater gains – but can also undertake greater risks. Again, more power to them – ***if*** they're right.

The fundamental breakthrough has been made; the methodology of the present study has been shown to generate very attractive annualized minimum and median returns.

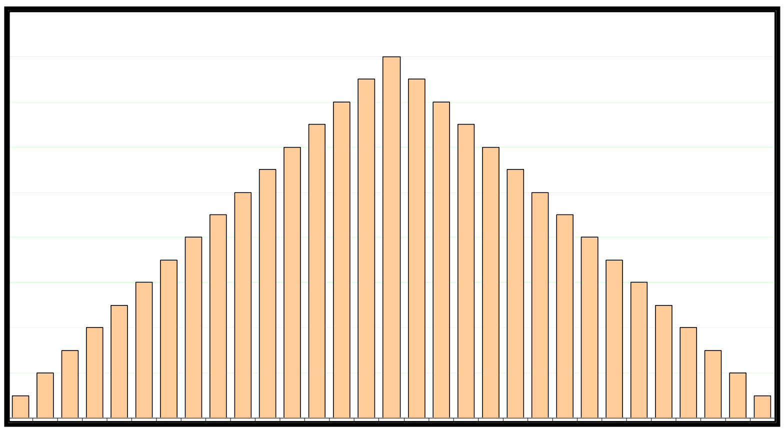
NEW SYNTHETIC INSTRUMENTS AND MEASURES

One final part of this paper is the construction of a great variety of synthetic ***integral instruments.***

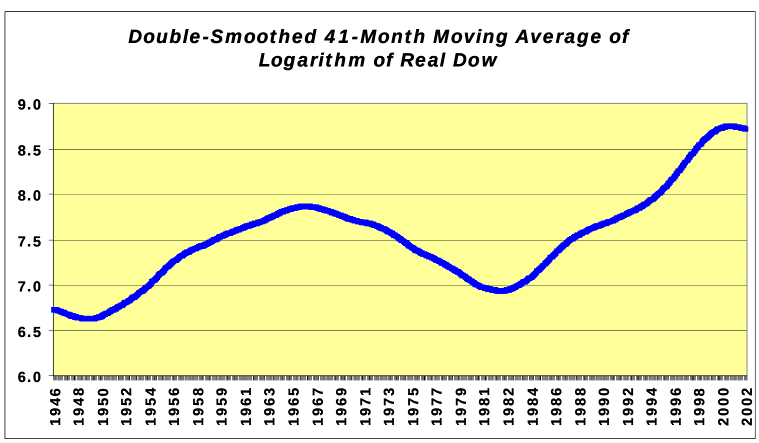
The integral investment measures described earlier in this paper are, properly speaking, not individual securities or financial instruments of their own. Rather, they are ***distributions*** or ***measures*** of buying followed, in the preferred and simplest form of the method, by an identical plan of selling after a time lag.

It is now possible to invest in "index futures" for many major markets. The present work goes well beyond existing structures to envision distributions or ***measures over time*** of these indices, chosen by mathematical sonar to fit the homeostatic cycles of major markets, transformed into new instruments or securities.

In particular, I envision ***integral measure instruments*** chosen to follow the major homeostatic cycles (rather than straddle them as do long-term retirement investment measures). For example, a double-smoothed 41-month integral security of the Dow (before or after inflation) looks like this:



Here the initial (small) purchases, measured in "units of the Dow," start in a certain month, rise to a peak 41 months later, and then diminish over the following 41 months. As has been seen, the graph of the value of these measures follows the long-term homeostatic cycle:



The graph above describes the rise and fall of the double-smoothed 41-month average in ***natural logarithm*** terms, so that an increase of 1.0 in the natural logarithm corresponds to a multiplication in value of $e = 2.718$, or 271.8% as high as before.

Getting past the technical terminology, it is clear from the graph that the 41-month triangular pattern of purchases (which we usually envision as followed by an identical time-lagged pattern of sales) tracks the long homeostatic cycle after inflation.

These shorter measures, ***whether presented as measured plans, or packaged up as synthetic financial instruments of their own,*** track the broad movements of the market over years, and smooth over the daily and monthly variations of the market. This represents an improvement over existing financial instruments.

Imagine that an investor in December 2007 buys a conventional "Dow future" dated December 2017. This would indeed reflect the changes in the Dow average over the intervening ten years. But the return would also be distorted, perhaps severely, by the short-term or seasonal variations in and immediately around the "closing month" of December 2017. In fact, if the future was pegged to a certain day, it would reflect the motions of the market on that very day! On the starting end, the value of the future would also be distorted by the vagaries of the market in and around the "starting month" of December 2007.

In contrast, an integral measure – or, if packaged together as a new security by itself, an ***integral instrument*** – would reflect the ***average*** or ***typical*** level of the Dow for many months ***around*** December 2017. Dividends could by arrangement be bundled into the security or separated from it – either possibility having its own effect upon the price.

In the preferred form of the present method, and in the discussion in the following paragraphs, a double-smoothed (triangular) distribution is used to generate these integral instruments. In alternative forms, any of the other distributions described in this paper and in its supporting spreadsheets, or variations built from them, with different possible functional, parametric, and genetic changes, could also be used.

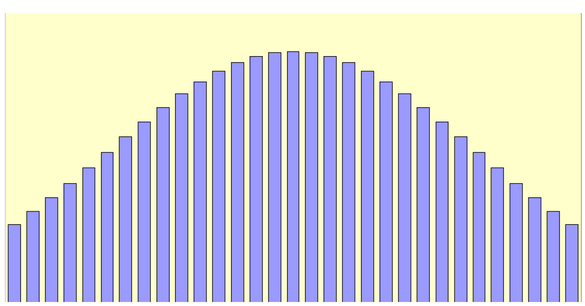
A ***push-pull arrangement*** with one triangle centered in December 2007 and the other centered in December 2017 – buy the first triangle, sell the second – would reflect the ***overall ten-year change of the index,*** rather than being distorted by the abnormalities of the opening and closing days and months. Isn't this what an investor would really like to follow? This can be packaged as a ***push-pull security*** which reflects the ***difference*** in value of the two triangles over the ten years – before or after inflation.

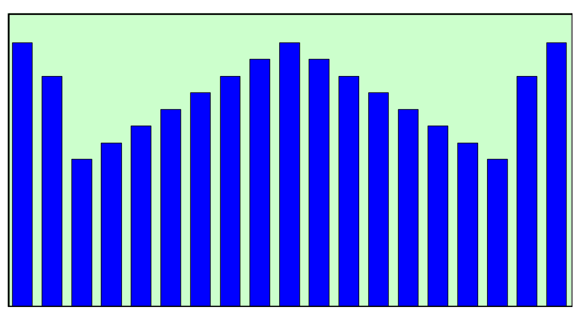
Since daily and monthly variations in the market can be considerable in size, this arrangement removes a great deal of uncertainty and allows an investor to better ***invest in the true direction of the market.*** One can never know what might happen on a given day, or during a given month or even during a given year. However, it is often possible to know [or believe that one knows] if the market is in a bull or a bear phase. These integral measures or integral instruments would track that broad bull or bear market motion.

These integral instruments can be designed with even shorter terms in mind. For instance, a twelve-month double-moving average will track where the market "is" after smoothing out ***yearly*** or ***seasonal*** variations, but retain both the long-term homeostatic cycle and any 4-year shorter-cycle motions.

Of course, these constructions can be designed for other major markets, such as the S&P 500 index, major market indices of other large nations, national income indices, real estate price indices for major markets such as the HPI index, and many other data series.

And these integral measures (or integral instruments packaged and sold as securities of their own) need not be limited to rectangular or triangular form. They can also be designed following sinusoidal, bell-shaped, truncated triangle, consolidated triangle, or other patterns, including but not limited to the patterns presented in this paper and its supporting Excel files, or perhaps specially designed by a genetic-algorithm process of adjustment. See the pictures below for two possible examples.





Moving averages (single or double) cannot be known in current time. It would involve a time lag to exactly find out what an integral instrument turned out to be "worth." The final "settlement day" would be some time later than the center of the second triangle. Parties could make a mutual agreement as to how to account for this extra "tail time," perhaps multiplying the "tail time" by the current Treasury-bill interest rate at the center of the selling triangle. Or, it might be advisable to truncate the tails of a triangle or other distribution to shorten the time involved.

Even the ***current*** moving average (the initial triangle or rectangle) cannot be known in current real time. One solution is to push the "starting time" 41 months (or so) back into the past, so that the initial triangle's value ***is*** completely known at the start. A corresponding "future" or "push-pull" arrangement might reflect the value of such a triangle ***ending*** in December 2017 – whose value is completely known at ***that*** time, ***minus*** (or divided by) the value of the corresponding triangle ***ending*** in December 2007, whose value is completely known in December 2007. This difference could be calculated based on the integral weighted summed values of the Dow triangles, before or after inflation. Alternatively, any slice of the two triangles could by agreement have its value brought "to current" through a price index or using an agreed-upon interest rate (perhaps a fixed rate or perhaps that of 10-year Treasury bills).

A useful alternative is the use of ***half-triangles,*** representing the first, or trailing, part of the distribution (the left half of the triangle). Those integral instruments ***can*** be valued on a current-time basis, since their value has already been defined (that part of the triangle is already in the past!). These half-triangles would not be as robust as the full triangles, but the loss in robustness might be balanced by the fact that the problem of time lag is avoided, they take less time to carry through than a full triangle, and all measures can be valued on a current-time basis. Of course, this halving need not be confined to triangles; one can work with other types of symmetric distributions as well.

One new and very important contribution of these special synthetic instruments is that they are ***more, not less, stable than their original "underlying" assets.*** This characteristic of "integral instruments" is the direct opposite of the instability of presently existing "derivatives." Because of this characteristic alone, integral instruments can become an important part of today's financial marketplace. The stability of these instruments, as with long-term retirement-oriented integral investment measures, would be a powerful force to "do good in the world."

Still another new and non-obvious feature of the present work is the construction of futures, options, and other securities built around ***phase time*** rather than ***real time.*** Imagine being able to purchase an instrument that expresses the value of a stock market index or other index – or better, one of its integrally chosen moving averages – ***at its peak in the current cycle – regardless of the time that peak actually happens, or how high or low that peak is.*** This would be a "maximum" security built around ***phase time.*** The investor would be making a statement about "how high the market will go" without any tight restriction on exactly what time that peak would actually be reached. There could be a maximum time limit (5, 10 or 20 years) placed on the contract so that no party could say the "current cycle" still had yet to run out or reach its peak. Conversely, securities could reflect a ***minimum*** of a market index (before or after adjusting for inflation; with or without a moving average distribution) measured by ***phase time.*** Other securities could measure the ***total difference between trough and peak values, whenever they happened to be attained.*** More possibilities are easy to imagine – let the mind run free!

These phase-time instruments would not be limited to the stock market, but could be constructed in the context of many major markets such as national income levels, commodity prices, or even synthetic constructions such as price/earnings or price/dividend ratios.

Furthermore, all of the investment measures and integral instruments described in this paper could themselves constitute the underlying asset values for a set of ***"new derivatives."*** These "new derivatives" would not be "derived" from a market index but from an integral instrument or investment measure. The integral instruments could themselves be "futured," "optioned," "hedged," "swapped," "sliced," "cross-hedged," and generally modified in various mathematically simple or exotic fashions, just as a veritable galaxy of "derivatives" exists today. These "new derivatives" would be less stable than the integral instruments from which they were derived, but still more stable than the "derivatives" which now exist. These "new derivatives" in turn constitute a novel and non-obvious feature of the present work.

Some of the integral instruments discussed so far remove or factor out the short cycle and thus track the long cycle up and down. Looking on a longer-term basis, it is also possible to build integral instruments, sold as separate securities, which follow not the long cycle, but the ***longer background trend***. Many studies have been produced showing that the Dow, with dividends reinvested, produces a long-term yield as high as 7 to 10 percent.

The difficulty here is that the Dow index must be held for several decades to pass through one or two complete long cycles. The Dow index can be falling in the long cycle for a decade at a time. Human biology makes a 50-year or 75-year investment security unattractive to an individual.

As we have seen, a double-smoothed 30-year (or 31-year, etc.) moving average, which will be called LONGBUY to give it a name, would smooth out this cycle and follow the long-term trend, avoiding the long cycle entirely. Such a security would catch the overall trend of growth ***in constant dollars, after inflation***, and ***without allowing for dividend reinvestment***. If an allowance were made for dividend reinvestment, presuming the stocks to be actually bought and held, by an institution if not by the individual investor, this true rate of return could be raised even further. Perhaps some tax deferral could be arranged where the profits and dividend reinvestments are not taxed until the security is finally cashed, in a manner analogous to what is done with retirement accounts, at which time it would be nice if the profits could be taxed at a favorable long-term capital gains rate instead of being taxed as ordinary income!

Such a double-smoothed integral instrument would have the advantage of being relatively immune from inflation, recessions and depressions. This steady background return (after inflation and without dividend reinvestment) would be realized year after year, right on through the great depression of the 1930's and the inflation and bear market of the 1970's. An investor in the possession of such securities could indeed sleep well at night. Of course, this investor would lose the advantage of getting in and out of the market at the bottom and top of the long cycle; on the other hand, he or she would not have to worry about timing the market, and could sleep well at night for decade after decade.

The greatest problem with these very-long-term integral measure instruments is the large time lag involved. A double-smoothed 30-year average requires 30 years after the "center time" to be exactly known. In 2007, we would just now be firmly determining the value of this long average centered in the year 1977!

The time-lag problem is somewhat mitigated by the fact that we expect these very-long-term securities to be stable over decades. Presumably they are to be considered as very-long-term investments of 50 years or more, which at first glance makes them unattractive to an individual investor, although not to governments or major firms.

The very stability of these special securities can be exploited to overcome the problem of time lag and make these securities into workable investments, thus truly capturing the attractive very-long-term return on the Dow.

Suppose that an investor in 2007 bought a 30/60-year future of LONGBUY. (Since the rate of return can be reasonably estimated, there is no wild uncertainty in pricing these futures or offering them for sale as bonds are sold.) The dual nature of the "30/60" must now be explained. Here it is intended that the security will be cashed out in 60 years, in 2067, when the double-smoothed 30-year moving average ***for (centered on) 2037***, 30 years into the future, finally becomes known. Now, it is important to define a fair carry-forward arrangement. Probably the best thing to do is to simply extend the rate of return realized by LONGBUY from 2007 to 2037 for an additional 30 years until the cash-out time of 2067.

Alternatively, an investor in 2007 can purchase a 30/60 year ***option*** on LONGBUY, which can be exercised in 2067 at a pre-defined price.

As a more interesting idea, an investor in 2007 can purchase a 60-year ***push-pull*** on LONGBUY. In 2007 the value of LONGBUY ***for (centered on) 1977*** is known. This 60-year push-pull security would measure the change in LONGBUY ***from 1977 to 2037***, a 60-year time span, and would be cashed out ***in 2067, when LONGBUY for 2037 finally becomes known***, again after a 60-year time span.

The great problem here is again that of time lag. Unless the human lifespan comes to be radically extended, no investor will want to wait 60 years to be paid off. Even an investor who expects to retire at age 100 would tend to become impatient, because no one would enjoy waiting for the final "tail" of LONGBUY to be tallied when long before the final tip of the tail he or she would have an excellent idea of where

LONGBUY would wind up. Presumably such an investor would be willing to sell one's LONGBUY security years before final maturity, knowing to a high degree of accuracy what the return would ultimately be.

There are several answers to the problem of time lag. First, as has just been mentioned, an investor could sell the LONGBUY security years before the final cash-out date, knowing that the last few years of the "tail" would have only a modest impact on the ultimate cash-out value.

Second, LONGBUY instruments could be constructed using half-triangles, truncated or "short" triangles, or flat distributions, or some other distribution, not stretching over quite as long a time span. Also, half-triangles can be built terminating in a particular current year and not possessing any "future side" – so that the value of these instruments ***can*** be known on a current basis and does not have such a width in time.

Third, there is no reason not to invest in shorter-term futures, options, and push-pulls in LONGBUY. An investor in 2007 could purchase a ***10-year push-pull*** security in LONGBUY. This investment would track the increase in LONGBUY (in other words, would follow the very-long-term trend) ***from 1977 to 1987***. In 2007 LONGBUY is known completely for 1977. This security would be cashed out in 2017, after a more reasonable time span of 10 years, when LONGBUY for ***1987*** finally becomes known. An advantage to the investor here is that he already would have in 2007 a fairly good idea of where LONGBUY was headed, lacking only the "tail" from 2007 to 2017. The issue that part of the period from 1977 to 1987 was a bear market and part was a bull market is not important. The very definition of LONGBUY means that this security smoothes over such recessions and rises; remember that LONGBUY of 1977 will include information from the boom of the 1960's and some information from the boom of the 1980's, as well as recessionary information from the 1970's; while LONGBUY of 1987 will include recessionary information from the 1970's and also boom information from the 1980's and 1990's, plus something of whatever happens from 2007 to 2017.

This is a good answer to the problem of time lag. These securities in LONGBUY could be designed to be cashed out over relatively short time spans, such as 5 years, 10 years, or 15 years. These securities would be very stable because the investor and selling institution would already know a great deal about where LONGBUY was going. For instance, half the triangle (the "past half") going into LONGBUY for 2007 is already known in current time, thus considerably reducing the uncertainty concerning the cash-out value in 2037 of a security based on LONGBUY of 2007, when the other half of the triangle (the "future half") finally becomes known in 2037. ***Thus, these securities would tend to be very stable and uninteresting in their movements, but would constitute excellent long-term investments for governments and retirement funds because they would be both relatively recession-proof***.

Finally, the most useful remedy for the problem of time lag is the construction of a ***secondary market*** in very-long-term integral investments. The question can be asked, "Why would an investor be interested in waiting 30 or 60 years to cash out an investment? Few people will live that far into the future." The answer is simple: the investor can sell out the future, option, or push-pull security ***before final maturity, on the secondary market***. A future, option, or push-pull security to be cashed out in 2037 or 2067 can be sold early, in 2017 or 2027. By that time a large part of the total time span will have already passed. The security can then be sold prior to maturity at a price based on the balance between known returns already realized (from 2007 to 2017 or 2027) and the anticipated future returns (from 2017 or 2027 until the final cash out). This procedure would be very similar to the present market in ***bonds*** or ***government debt*** or ***mortgage-backed securities;*** these instruments are frequently traded long before they actually mature rather than simply being held until the final due date. Alternatively, these securities can be sold at any time and simply "rolled over" into similar securities maturing an additional ten or twenty years down the line, in a manner analogous to the way the government finances its national debt by simply exchanging it for more debt.

Of course, profits and losses can occur in this secondary market. An investor who guesses the long-term growth rate more accurately than the selling institution can realize a substantial profit, just as in the case of a bond investor who correctly foresees the direction of interest rates, or a wise (or fortunate) investor in futures or options. However, these securities will be more stable than bonds because they are already relatively recession-proof and take a "long view."

To trade in this secondary market will require wisdom. At the very least, though, traders in this market will not have to pay overwhelming attention to daily news events, because the securities are so long-term and stable. In fact, it is probably advisable ***not*** to pay too much attention to these short-term events. And a wise investor can reap substantial profits if the market in fact ***was*** to pay too much attention to short-term events, as is the human tendency; simply follow a "Buffett-style" value-investing strategy of buying when the market foolishly undervalues these securities and selling when the market overvalues them. Such a strategy is fairly workable because these securities are so long-term that their value will be dominated by fundamental considerations rather than technical short-term variations, and if the market in these securities ***does*** experience short-term gyrations, these variations are almost certainly unjustified and can be taken advantage of by an investor with a fundamental point of view.

The existence of such a secondary market will reintroduce the old balance of wisdom and power between buyer and seller. How should a very-long-term security be valued? What will the final cash-out value be, and what model should be used to estimate that value? What pricing arrangement should the selling institution offer to the investor? ***Should the investor be charged a premium by the selling institution because of the stability and the almost recession-proof nature of these securities?*** This could be very lucrative indeed. ***In summary, these securities have finally captured the very-long-term high return of major markets. The issue now is simply how to divide this additional monetary advantage between investor and selling institution***.

In recent decades, money has been strongly flowing into the market, particularly into mutual funds, especially as people of the "baby-boom" generation born from 1946 to 1964 at long last wake up and invest for their retirement. I myself am part of this generation, having been born in 1953. But there is no reason why this money cannot also flow ***out of*** mutual funds and ***out of*** the stock market in general.

The cycle analysis of this paper, including the Phase Method, concurs strongly with what is known about economic and demographic fundamentals. The "baby boom" generation is due to start retiring in approximately 2010. The last baby boomers will retire at about 2030. If these people live in retirement for 10 to 20 years before death, they will probably be drawing on their funds all through this period. Just as the flow of money ***into*** the market has recently pushed prices ***up***, so will the necessary flow ***out of*** the market likely act as a prolonged ***downdraft*** on the market. Many baby boomers will find themselves highly disappointed when they seek to sell out, getting only a fraction of what they hoped for.

Also concurring with this conjecture is the fundamental problem of the pile-up of unpayable ***entitlements*** that will come home to roost. The federal government already spends much of its income on entitlements and interest on debt. Social Security is not properly funded for the future, and neither are Medicare entitlements. At some point the inevitable day of reckoning will arise when it becomes impossible to pay all the entitlements that are due and expected. Possibly an answer will be found in the form of inflation, until recently almost forgotten, where everyone "gets" the entitlements they expected in current dollars, which buy only a fraction of what they were expected to buy. In such a case the stock market could rise much higher than its present levels, as measured in ***current dollars,*** while actually falling in ***constant dollars.*** The "stagflation" of the 1970's could return.

A deep bear market – or a "stagflationary" period – would be a sad time for all the people of my generation. However, it could be a good time for my two children, born in 1993 and 1995, to invest. Just as economic booms pass, so do times of difficulty. Just as night follows day, so also does morning follow night.

In conclusion, the subjects of homeostatic cycle analysis and phase navigation provide insight in understanding the present and future performance of major markets. The construction of integral investment measure plans and integral instruments or securities produces a useful way to insulate an investment against inflation and against the recessionary periods of the different cycles in the market, thus capturing the long-term rate of return of a major market. These synthetic investment instruments or securities could themselves constitute a major market offered to investors, and could prove lucrative to investors and selling institutions alike.

In summary, the present work has many new, useful, helpful, and potentially valuable features. This paper is respectfully submitted for the reader's perusal.